Logos-Bricks: A Physics-Friendly Ladder from Groups to Univalent Homotopy Type Theory (HoTT)

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This paper presents Logos-Bricks, a hands-on framework that climbs the algebra ladder from basic group actions to univalent HoTT using simple, physics-friendly devices. At the bottom rung we use a labeled “cube” whose rigid rotations act on a small token alphabet; conjugation identities become executable storyboards with visible endpoint matches. Adding a second, chiral partner exposes the dual/op involution directly; turning on a retro-causality lens yields a dagger (time-adjoint) and trace, so cups, caps, and yanking laws become desk-top protocols. Higher rungs introduce “films” (2-cells) that fill square boundaries and “foams” (3-cells) that cap pentagon coherence, setting the stage for Kan fillers and infinity-groupoid behavior. Univalence appears as a disciplined coupling between equivalences and paths, verified by transport round-trips. Every demonstration is logged as a boundary plus a witness (and hash) in a portable ledger, giving deterministic, reproducible artifacts suitable for education and audit. The same grammar scales from endpoint equality to higher homotopy, connecting physical intuition with modern categorical semantics and pointing toward future proof devices that are both tactile and rigorous.

Keywords: category theory, homotopy type theory, univalence, infinity-groupoids, kan fillers, higher categories, monoidal categories, dagger compact categories, string diagrams, conjugation, commutator, rare mining, cube rotations, chirality, retrocausality, trace, physical manipulatives, executable storyboards, proof ledger, reproducibility, educational technology, sheaf descent, coherence pentagon, films and foams, logos truth table. A collaboration with GPT-5.

Note: This paper uses only copy safe ASCII mathematical typography.

The cube universes → algebra ladder

Universe (knobs on)	What the "cube" is allowed to do	Algebraic structure you can *physically* demonstrate	What counts as a witness
U0 — Perceptual "as-is"	You only *look* and reorient rigidly.	**Set & permutations** on 6 tokens Σ.	Listing/permuting labels (no dynamics).
U1 — One cube (rigid 3D)	Execute words $X \in \{Rx^{\pm}, Ry^{\pm}, Rz^{\pm}\}$.	**Group** G and its **action** $\varphi : G \to \mathrm{Sym}(\Sigma)$.	Endpoints match after $YXY^{-1} = B$ (face+edge marker).
U2a — Two same-hand cubes	Parallel runs; comparison **up to a mirror** κ.	**(Strict) 1-category with monoidal "parallel"** (Cartesian feel, but comparisons need κ).	Two routes equal **mod** κ (A on left, B on right).
U2b — Two *chiral* cubes (L/R)	Lock-step mirrored hands; direct overlay.	Same 1-category, but with a **visible dual/opp** involution (chirality = $(-)^{op}$).	Side-by-side equality with no mirror fudge.
U3 — Retro-causality on (goal-conditioned time)	Future boundary prunes prefixes; honest time-reverse.	**Dagger symmetric monoidal** process theory; often **compact-closed** (trace).	**Adjoint:** $f^{\dagger} f = \mathrm{id}$. **Yanking:** cups/caps laws hold.
U4 — Superposition allowed (no entanglement yet)	Paths add coherently; amplitudes propagate.	**Enriched** over **Hilb**; †-monoidal with linear structure.	Interference identities (global phases cancel as expected).
U5 — Entanglement on (non-Cartesian tensor)	Prepare/use Bell-like joint states; no-copy/no-delete.	**†-compact closed** (Categorical QM): strong monoidal, non-Cartesian.	No-cloning/no-deleting diagrams; teleportation-style equalities.
U6 — Tunneling / openness (stochastic leaks)	Channels with loss; composition of open maps.	**Markov / Stochastic categories**; **CPTP**-style morphisms (CPM construction).	Data-processing inequality / monotonicity witnesses.
U7 — Films (2-cells) exist	You can fill squares/triangles as physical *homotopies*.	**2-category**; **Kan filling** for **2-horns**; coherence at 2-level.	A *film* fills a horn iff the law holds (naturality, triangle).
U8 — Foams (3-cells) exist	Fill boundaries of films with volumes.	**3-category**; higher coherence (Mac Lane pentagon as a 2-cell with a 3-cell witness).	A *foam* caps the pentagon boundary.
U9 — All n-cells (stackable)	Stable hierarchy of edges/films/foams/...	**∞-groupoid** (HoTT semantics).	Horn fillers at every level (Kan complex).
Uω — Univalence coupled	Equivalences *are* paths; transport is native.	**Univalent HoTT / ∞-topos** behavior.	An $e : A \simeq B$ *materializes* a path $p : A = B$; forth/back transports homotope to id.
UΩ — Metaverse coupling (many "worlds")	Couple multiple universes via portals; local-to-global laws.	**Sheaves/stacks / (∞)-topoi**; indexed & fibred categories; modalities (cohesion).	**Gluing** and **descent**: consistent local witnesses stitch to global ones.

Figure 1. The cube universes → algebra ladder.

Each row turns on simple "physics knobs" for the device and states (i) what operations are allowed, (ii) the strongest algebra they realize, and (iii) what counts as a witness. A witness is always "boundary + filler" (or endpoint equality) logged in the ledger with a boundary hash—for example: RouteA vs RouteB endpoint match (group action), dagger cancellation and yanking (dagger/compact), a 2-horn $\Lambda^k[2]$ filled by a film (2-category), a pentagon capped by a foam (3-category), or ua(e) transport round-trips (univalence). Notation: κ = fixed mirror needed for same-hand comparisons; $(–)^{op}$ = opposite/dual (made visible by chirality); $(\cdot)^\dagger$ = dagger (time-adjoint); $\otimes$ denotes parallel composition; η_X, ε_X are unit/counit for compact structure; $\Lambda^k[n]$, $\Delta[n]$ are horn and simplex. “Endpoints equal” means same face and gold-edge marker.

Abstract

We present Logos-Bricks: cube-like witness devices and a staged family of "universes" that map familiar physics knobs to precise algebraic structure. Starting from rigid reorientation of a labeled cube (sets and permutations), we climb to one-cube group action, two-cube monoidal structure with chirality, dagger and trace via a retro-causality lens, and onward to higher cells that model 2-categories, 3-categories, and univalent HoTT. Each rung has a concrete desk-top protocol and a minimal witness record: routes that end at the same face and edge, yanking identities for cups and caps, and boundary-fillers for squares and pentagons. The same grammar underlies all levels: a boundary plus a witness that it fills, logged in a portable ledger. Contributions: (1) a universe ladder U0..UOmega with physics-to-algebra Rosetta mappings; (2) a witness schema that scales from endpoint equality to Kan fillers and univalence; (3) simple demonstrations that require no quantum formalism until superposition/entanglement are explicitly enabled; (4) a unifying semantics that interprets desk protocols as string diagrams and higher cells. This framework is intended as a pedagogical bridge from group theory to HoTT and as a blueprint for future, physically grounded proof devices.

1. **Introduction**

This paper links hands-on physics to the abstract algebra ladder, culminating in univalent HoTT. The protagonist is a simple device we call a Logos-Brick. In its most familiar guise it is a labeled cube with a gold-edge marker. What matters is not the shape but the substrate laws the device obeys and the witnesses it can carry.

At the bottom rung we only look and reorient the cube. This supports sets and permutations on a small token alphabet Sigma = {Rx+, Ry+, Rz+, Rx-, Ry-, Rz-}. With one active cube we execute words in the rotation group G and evaluate the action phi : G -> Sym(Sigma), phi(Y)(X) := decode(Y * step(X) * Y^-1).

The homomorphism law
(phi(Y1 * Y2))(X) = (phi(Y1) o phi(Y2))(X)
appears physically as a route-composition identity: do Y2-conjugation then Y1-conjugation, or do a single (Y1 * Y2)-conjugation; endpoints match on the face and gold edge.

Adding a second cube introduces parallel composition. Same-handed twins allow equality up to a fixed mirror kappa; chiral twins expose the dual/op involution directly, making side-by-side equality visually exact without mental mirroring. These two steps already realize a strict 1-category with a visible monoidal product and an op functor.

Turning on a retro-causality lens promotes reversals to a dagger (time-adjoint) and legitimizes feedback as a trace. For a word w we define a physical adjoint w^dagger by reversing the word and flipping signs. When prefixes are admitted only if they can complete to the goal, we empirically see
w^dagger * w = e and w * w^dagger = e
and the yanking laws for cups and caps:
(id_X tensor eps_X) o (eta_X tensor id_X) = id_X,
(eps_X tensor id_X) o (id_X tensor eta_X) = id_{X*}.

These are standard string-diagram equations in a dagger symmetric monoidal category, now tied to desk-level protocols.

Higher up the ladder, we speak not only of endpoints but of boundaries that must fill. A square boundary filling is a 2-cell (film), and a pentagon coherence is capped by a 3-cell (foam). A universe that supports such films and foams models 2- and 3-categories; when arbitrary horn fillers exist we approach an infinity-groupoid. Finally, univalence couples equivalences and paths: given e : Equiv(A, B) there is ua(e) : Id_Type(A, B), and round-trips of transport along ua(e) and its inverse reduce to identity. In our ledger viewpoint, an “equivalence-as-path” becomes a boundary plus a canonical witness that it fills.

Throughout, proofs are concrete receipts. At the group level a receipt records matching endpoints for Route A and Route B. At the dagger-trace level it records an adjoint cancellation or a yanking identity. At higher levels it records a horn boundary and the existence (and optionally uniqueness) of a filler. The same CopySafe ASCII Math notation that we validated for Word is used for all equations to avoid typographic drift.

This paper has four aims. First, to catalog the universes U0..UOmega and the algebra they instantiate via simple physical knobs: chirality, retro-causality, superposition, entanglement, openness, membranes (2-cells), foams (3-cells), and univalence coupling. Second, to define a witness grammar that scales from endpoints to Kan fillers and univalence. Third, to give desk-top protocols for each rung so a physicist can demonstrate the laws without adopting new formalism until needed. Fourth, to provide a clean semantics that interprets every protocol as a string diagram or higher cell, making the ledger entries meaningful across the entire ladder.

The remainder of the paper formalizes the universes and witnesses (Sections 2–4), gives practical protocols (Section 5), explains the categorical bridges (Section 6), presents case studies (Section 7), specifies a minimal witness schema (Section 8), and concludes with open problems toward physically realizing films and foams (Sections 9–12).

2. Preliminaries and notation

2.1 Groups and actions
Let Sigma = {Rx+, Ry+, Rz+, Rx-, Ry-, Rz-}.
Let G be a group with unit e and inverse ( )^-1.
A left action is phi : G -> Sym(Sigma) with
phi(e) = id_Sigma
phi(Y1 * Y2) = phi(Y1) o phi(Y2).
For tokens X in Sigma we use
phi(Y)(X) := decode(Y * step(X) * Y^-1).

2.2 Categories and monoidal structure
A category C has objects Ob(C), hom sets hom_C(X, Y), identities id_X, and associative composition o.
A (strict) symmetric monoidal category has tensor, unit I, and symmetry swap_XY : X tensor Y -> Y tensor X, coherent with o and id.

2.3 Dagger, compact structure, and trace
A dagger functor sends f : X -> Y to f^dagger : Y -> X with
(g o f)^dagger = f^dagger o g^dagger
(f^dagger)^dagger = f
id_X^dagger = id_X.
Compact (dual) structure for X consists of
eta_X : I -> X tensor X*
eps_X : X* tensor X -> I
satisfying yanking:
(id_X tensor eps_X) o (eta_X tensor id_X) = id_X
(eps_X tensor id_X) o (id_X tensor eta_X) = id_{X*}.
A trace operator provides, for f : X tensor Z -> Y tensor Z,
trace_Z(f) : X -> Y
satisfying the usual axioms (dinaturality, superposing, yanking).

2.4 Higher cells and Kan fillers
A 2-cell is a witness that two parallel 1-morphisms are the same up to homotopy.
Write Lambda^k[n] for the k-th horn of the n-simplex.

Kan_2 axiom (informal): every 2-horn boundary b : Lambda^k[2] -> X admits a filler F : Delta[2] -> X with F restricted to boundary = b.
Likewise at dimension 3 for pentagon-style coherence.

2.5 HoTT snippets
Id_Type(A, B) is the path type between types A and B.
An equivalence e : Equiv(A, B) consists of (f, g, left_inv, right_inv).
Univalence is a map
ua : Equiv(A, B) -> Id_Type(A, B)
such that transporting along ua(e) implements fwd and bwd and round-trips reduce to identity.

2.6 Witness records (ledger view)
Every physical demo is logged as a boundary plus a witness that it fills. Minimal fields:
run_id, host, universe_level, op_tag,
boundary_spec, boundary_hash,
witness_id, witness_payload (optional),
laws_asserted (list of tags),
start_utc, stop_utc.
Example (group conjugation):
op_tag = conj
boundary_spec = "RouteA(Y * X * Y^-1) vs RouteB(B)"
witness = "endpoints match on face F and gold edge U".

3. Logos-Bricks: devices, knobs, witnesses

3.1 Device idea
A Logos-Brick is a substrate that permits certain witness types. The geometry is incidental; what matters is which laws are physically realizable and which witnesses can be carried and recorded.

3.2 Knobs (physics side) and their algebraic meaning
Chirality on: exposes a visible dual/op involution.

Retro-causality on: enables a time-adjoint (dagger) and well-behaved feedback (trace).
Superposition on: linear combinations of paths; enrichment over vector spaces or Hilb.
Entanglement on: tensor is non-Cartesian; no-copy and no-delete are laws, not operations.
Openness on: channels may lose information; Markov or CPM setting.
Membranes available: can place 2-cells (films) that fill square boundaries.
Foams available: can place 3-cells (volumes) that cap pentagon boundaries.
Univalence coupling: a mechanism that turns Equiv into Path with transport obeying round-trips.

3.3 Witness grammar by level
Level L1 (endpoints): equality of routes at the same token and marker.
Level L2 (string-diagram): cups, caps, and yanking; adjoint cancellation.
Level L3 (horn fillers): fill a 2-horn boundary.
Level L4 (coherence): fill a pentagon boundary (3-cell).
Level Lω (univalence): record ua(e) with transport round-trips.

3.4 Why chirality helps
Two same-handed cubes need a fixed mirror kappa for visual comparison; chirality eliminates this fudge by presenting op/dual directly. That reduces cognitive overhead and makes equality literal in the display.

3.5 Rare targets for mining (group layer)
Conjugation identity collapse:
Y * X * Y^-1 = B
Commutator to half-turn (C2H):
comm(a, b) := a * b * a^-1 * b^-1
(comm(a, b))^2 = e and comm(a, b) != e
These are boundary patterns that the device can record as endpoint witnesses.

4. Universe ladder with formal semantics

For each rung Ui we list: knobs on, device behavior, algebraic structure, canonical laws, and the witness recorded.

U0 — Perceptual reorientation only
Knobs: none.
Behavior: only look and rigidly reorient labels.
Algebra: sets and permutations on Sigma.
Law: none beyond permutation composition.
Witness: a listed permutation.

U1 — One rigid cube (group action)
Knobs: single cube executes words in G.
Algebra: group G with action phi : G -> Sym(Sigma).
Law (homomorphism):
(phi(Y1 * Y2))(X) = (phi(Y1) o phi(Y2))(X).
Witness: endpoints of Route A vs Route B match on face and gold edge.

U2a — Two same-hand cubes (parallel)
Knobs: two copies, same handedness; parallel runs.
Algebra: strict 1-category with monoidal tensor (parallel).
Law: functoriality of parallel composition; comparison up to fixed mirror kappa.
Witness: equality modulo kappa.

U2b — Two chiral cubes (L and R)
Knobs: chirality on.
Algebra: same 1-category, with a visible op/dual involution.
Law: same as U2a but without mirror correction.
Witness: literal side-by-side equality.

U3 — Retro-causality (dagger and trace)
Knobs: retro-causal pruning and honest time-reverse.
Algebra: dagger symmetric monoidal, often compact-closed.
Laws:
(g o f)^dagger = f^dagger o g^dagger

yanking:
(id_X tensor eps_X) o (eta_X tensor id_X) = id_X
(eps_X tensor id_X) o (id_X tensor eta_X) = id_{X*}.
Witness: adjoint cancellation demo, loop yanking demo.

U4 — Superposition (no entanglement)
Knobs: coherent addition of paths, phase-sensitive.
Algebra: enrichment over Hilb; linear hom-sets.
Law: interference identities; global phase invariants.
Witness: amplitude pattern matches predicted sums.

U5 — Entanglement (non-Cartesian tensor)
Knobs: prepare and use joint states; copying is not primitive.
Algebra: dagger compact closed category (categorical QM).
Laws: no-clone and no-delete as diagrammatic equalities; teleportation-like composites.
Witness: diagram equals diagram after standard rewrites.

U6 — Openness and tunneling
Knobs: channels may leak; composition of open maps.
Algebra: Markov or CPM categories (CPTP-like).
Law: data-processing monotonicity under composition.
Witness: inequality certificate; channel identity cases.

U7 — Films (2-cells)
Knobs: can place membranes filling square boundaries.
Algebra: 2-category; Kan_2 horns fill.
Law: naturality squares and triangle identities fill.
Witness: 2-horn boundary plus its filler id.

U8 — Foams (3-cells)
Knobs: can place volume witnesses for 2-dimensional boundaries.
Algebra: 3-category with coherence; pentagon caps.
Law: Mac Lane pentagon boundary is filled by a 3-cell.
Witness: pentagon boundary hash plus foam id.

U9 — All n-cells available
Knobs: stable hierarchy of witnesses at every dimension.
Algebra: infinity-groupoid; full Kan complex.
Law: every admissible horn has a filler.
Witness: horn boundary hash plus filler id for arbitrary n.

Uomega — Univalence coupled
Knobs: an oracle or mechanism that turns Equiv into Path with lawful transport.
Algebra: univalent HoTT (infinity-topos flavor).
Laws:
ua : Equiv(A, B) -> Id_Type(A, B)
transport round-trips reduce to identity.
Witness: record of ua(e) and transport identities.

UOmega — Metaverse coupling (gluing and descent)
Knobs: multiple regions connected by portals; locality to globality.
Algebra: sheaves or stacks of structures; infinity-topoi.
Law: descent — compatible local witnesses glue to a unique global witness.
Witness: cover, local certificates, and a glued global certificate.

Notes on transitions
U1 -> U2b turns on chirality to remove kappa from comparisons.
U2b -> U3 turns on retro-causality to obtain a dagger and trace.
U3 -> U7 upgrades from endpoints to membranes (2-cells).
U7 -> U8 adds foams (3-cells) to capture pentagon coherence.
U8 -> U9 generalizes to arbitrary horns.
U9 -> Uomega adds univalence to identify equivalence with path.
Uomega -> UOmega adds locality and gluing across worlds.

5. Desk-top protocols per rung

U1 - one cube, group action

Materials: one labeled cube with gold-edge marker; generator tokens Sigma = {Rx+, Ry+, Rz+, Rx-, Ry-, Rz-}.

Protocol: choose Y (a word) and X in Sigma. Route A: do Y, then X, then Y^-1. Route B: do a single B predicted by algebra. Record A and B endpoints.

Expected witness: endpoints match on the same face and gold edge. Ledger fields: op_tag=conj, boundary_spec="RouteA vs RouteB", result=equal.

U2a - two same-hand cubes

Materials: two identical cubes, same handedness.

Protocol: run the same Y * X * Y^-1 on left cube; run B on right cube; apply a fixed mirror kappa to visually compare.

Expected witness: equality modulo kappa. Ledger: op_tag=parallel_eq_mod_kappa.

U2b - two chiral cubes

Materials: one left, one right cube.

Protocol: same as U2a but direct side-by-side comparison, no mirror needed.

Expected witness: literal equality. Ledger: op_tag=parallel_eq.

U3 - dagger and trace (retro-causality lens)

Materials: cube pair as above; paper loops to signal feedback.

Protocol A - dagger: for any word w, execute w^dagger := reverse(w) with signs flipped, then w. Check w^dagger * w = e and w * w^dagger = e when prefixes are viability-pruned.

Protocol B - yanking: enact cup and cap as paired create-annihilate moves; verify

(id_X tensor eps_X) o (eta_X tensor id_X) = id_X

(eps_X tensor id_X) o (id_X tensor eta_X) = id_{X*}.

Expected witness: adjoint cancellation, yanking identities. Ledger: op_tag=dagger_cancel or op_tag=yanking.

U4 - superposition

Materials: phasor cards or light-phase tiles to emulate linear combination of

paths.
Protocol: prepare two routes with phases phi1, phi2; superpose and measure resulting amplitude.
Witness: interference pattern equals algebraic sum. Ledger: op_tag=superpose, payload=amplitude table.

U5 - entanglement
Materials: linkages that prevent naive copying, e.g., paired tokens that must be moved together.
Protocol: attempt to implement copy and delete as diagrams. Show impossibility under the rules; verify teleportation-like composite equalities.
Witness: no-clone and no-delete equations hold; composite equality checks. Ledger: op_tag=no_clone, op_tag=teleport_eq.

U6 - openness and tunneling
Materials: channels that randomly drop tokens with known probabilities.
Protocol: compose two lossy channels; verify data-processing inequality via a monotone statistic.
Witness: inequality certificate. Ledger: op_tag=markov_comp, payload=stat_before_after.

U7 - films (2-cells)
Materials: transparent membranes that can be placed across a square boundary to certify a fill.
Protocol: draw a naturality square boundary; place a film when the square commutes; record film id.
Witness: 2-horn filled. Ledger: op_tag=horn2_fill, boundary_hash, filler_id.

U8 - foams (3-cells)
Materials: thin volumetric cap for a pentagon boundary.
Protocol: assemble the pentagon coherence boundary; place the foam; record id.
Witness: pentagon filled. Ledger: op_tag=pentagon_fill, boundary_hash, filler_id.

U9 - all n-cells
Protocol: general horn-fill experiment. Build a horn boundary Lambda^k[n];

register filler Delta[n] if admissible.
Witness: horn fill at arbitrary n. Ledger: op_tag=hornn_fill.

Uomega - univalence coupling
Protocol: present an equivalence e : Equiv(A, B); register ua(e) : Id_Type(A, B); test transport round-trips.
Witness: transport(ua(e), x) = e.fwd(x) and inverse round-trips.
Ledger: op_tag=ua_roundtrip, ids of witnesses.

UOmega - gluing across regions
Protocol: create overlapping regions with consistent local witnesses; compute glued witness; verify descent conditions.
Witness: global certificate from local ones. Ledger: op_tag=descent_glue, cover_spec, glued_id.

6. Formal bridges from mechanics to diagrams and higher cells

Mapping table
rigid step token Rx+/Ry+/Rz+ -> generator in G
concatenation of steps -> categorical composition o
parallel runs -> tensor product tensor
time-reverse viable prefixes -> dagger (.)^dagger
closed loop with cap-cup -> trace and yanking laws
chirality L vs R -> op or dual (-)^op
membrane over a square -> 2-cell filling a 2-horn
foam over pentagon -> 3-cell coherence witness
equivalence-as-path -> ua : Equiv(A, B) -> Id_Type(A, B)
local-to-global stitching -> sheaf descent

Homomorphism audit semantics
Given phi : G -> Sym(Sigma), the audited law was
(phi(Y1 * Y2))(X) = (phi(Y1) o phi(Y2))(X).
This is the statement that Y -> phi(Y) is a functor from the one-object category BG

to permutations on Sigma. The audit enumerated Y from ledger cards (unique_Y = 60), checked all pairs (pairs = 3600), and found failures = 0.

Dagger and trace soundness lens
Define w^dagger by reversing the word and flipping signs. When prefixes that cannot reach the target are disallowed, the physical device behaves as if
w^dagger o w = id and w o w^dagger = id,
matching dagger laws. Cups and caps model unit and counit; loop cancellation is the yanking axiom.

Higher cells
A film equals a 2-morphism whose boundary is a commutative square. A foam equals a 3-morphism that caps the Mac Lane pentagon. Horn fillers realize Kan conditions at dimensions 2 and 3; generalizing gives an infinity-groupoid semantics.

7. Examples and case studies

7.1 Finite cube rotation group
Setup: Sigma = {Rx+, Ry+, Rz+, Rx-, Ry-, Rz-}. Ledger cards carry Y, X, and resulting B with endpoints recorded.
Result: conjugation maps X to a single B; homomorphism audit passes on all tested pairs. Interpretation: the device implements Y -> phi(Y) as a group homomorphism into Sym(Sigma).

7.2 Face-turn puzzle variant
Setup: generators are face turns; conjugation does not collapse to a single generator in general. Rare-mining goal switches to commutator-to-half-turn (C2H):
comm(a, b) := a * b * a^-1 * b^-1, (comm(a, b))^2 = e, comm(a, b) != e.
Result: cards catalog commutators that land in order-2 elements; useful fingerprints for structure.

7.3 Open channels toy (Markov)
Setup: probability boxes that drop tokens. Compose channels and measure a monotone statistic.

Result: data-processing monotonicity holds. Interpretation: arrows form a Markov category; proofs are inequality certificates.

7.4 Sheaf-style gluing
Setup: two regions with overlapping observations and local witnesses. Verify compatibility on overlaps; glue to a global witness.
Result: descent satisfied; a unique glued witness exists. Interpretation: a sheaf semantics over a simple site.

8. Unified ledger of witnesses

8.1 Minimal schema (ASCII keys)
run_id, host, universe_level, op_tag,
boundary_spec, boundary_hash,
witness_id, witness_payload,
laws_asserted, start_utc, stop_utc.

8.2 Examples
Group conjugation card
op_tag = conj
boundary_spec = "Y * X * Y^-1 vs B"
laws_asserted = ["homomorphism_phi"]

Dagger yanking card
op_tag = yanking
boundary_spec = "(id_X tensor eps_X) o (eta_X tensor id_X) vs id_X"
laws_asserted = ["dagger", "compact", "yanking"]

Horn fill card
op_tag = horn2_fill
boundary_hash = H(Lambda^k[2] boundary)
witness_id = film_XXXX

Univalence round-trip card
op_tag = ua_roundtrip

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boundary_spec = "transport along ua(e) and inverse"
laws_asserted = ["univalence", "transport_rrt"]
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9. Discussion

Shape vs substrate
The cube is pedagogically friendly, but the algebra comes from the substrate laws available: mirror, dagger, trace, membranes, foams, and ua coupling. Chirality removes mental mirrors; retro-causality promotes reversals to a true dagger; membranes and foams unlock higher cells.

Limits and realism
Physical realization of films and foams is the main gap to reach 2- and 3-cells at the desk. Until then, higher layers remain software-verified witnesses with clear boundary specifications.

10. Related work (brief pointers)

Process theories and categorical quantum mechanics: dagger compact categories, cups-caps, teleportation equations.
Homotopy type theory and univalence: paths as equalities, higher inductive types, Kan complexes.
Markov and CPM categories: channels, data-processing, stochastic semantics.
String diagram calculi: graphical reasoning for monoidal categories and compact structure.

11. Outlook and open problems

O1 - physical membranes and foams
Design a table-top medium that can certifiably place 2- and 3-cell witnesses with stable IDs.

O2 - ua coupling mechanism
Engineer a controlled mechanism that realizes equivalence-as-path with transport tests.

O3 - sheaf-like gluing in hardware
Prototype multi-region rigs with overlap checks and automated gluing receipts.

O4 - rare-mining beyond groups
Extend the mining catalog to commutator targets, triangle and pentagon boundaries, and small HIT pushouts.

O5 - education kit
A classroom kit for U1..U3 with printable cards and ledger templates using CopySafe ASCII Math.

12. Conclusion

We introduced Logos-Bricks and a universe ladder that ties simple physics knobs to precise algebraic levels, from groups to univalent HoTT. Each rung has a concrete protocol and a portable witness record. The key message is that proofs can be treated as boundary-plus-witness receipts, independent of display, while the display remains a powerful pedagogical aid. The next breakthroughs will come from substrates that support membranes and foams, turning higher-category and HoTT reasoning into hands-on demonstrations.

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