Invariant Overlap and the Birth of New Mathematics: A Source–Gaze Law in Admissibility Algebra

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New mathematics is neither passively found nor arbitrarily invented. It appears when latent structure in a mathematical source-field becomes visible under an admissible gaze. This paper formalizes discovery as a source–gaze interaction. Let Ω be a mathematical presentation space and π a receiver policy: a structured capacity to recognize invariants, reject artifacts, and form quotients. A mathematical object is born only when some invariant made available by Ω overlaps a recognitional invariant admitted by π. If the source is patterned, even a weak or wandering gaze may discover structure. If the source is random, only a patterned gaze can impose apparent structure, which then requires strict certification. If both source and gaze are random, no object forms. If both are patterned, discovery requires nonempty invariant overlap. This yields a compact admissibility principle: no new mathematics without invariant overlap; no theorem without certificate; no object without an admissible gaze.

Keywords: Admissibility Algebra, mathematical discovery, invariant overlap, admissible gaze, source–receiver duality, new mathematics, quotient object, proof certificate, artifact rejection, mathematical creativity, policy-relative identity, theorem formation, invariant recognition.

1. Presentation Space

Let Ω be a mathematical source-field. Its elements are presentations: diagrams, computations, examples, conjectures, formulas, simulations, categories, type-theoretic terms, graphs, or numerical traces. A presentation $X \in \Omega$ is not yet a mathematical object. It is pre-object material.

Objecthood requires admission under a policy π.

$X \mapsto [X]\pi$.

The quotient $[X]\pi$ is the mathematical object as seen under π.

2. Gaze

A gaze π is a structured receiver. It contains admissible invariants, transformations, rejection rules, proof standards, quotient rules, and certificate demands. It is not mere attention. It is the rule by which attention becomes mathematical recognition.

$\Gamma\pi : \Omega \to \Omega/\pi \cup R\pi$.

Here Ω/π is the policy-relative object-space, and $R\pi$ is the rejection or deferral space.

3. Invariant Overlap

Let $\mathrm{Inv}(\Omega)$ denote invariants latent or available in the source-field. Let $\mathrm{Inv}(\pi)$ denote invariants recognizable by the gaze. Discovery occurs only when:

$\mathrm{Inv}(\Omega) \cap \mathrm{Inv}(\pi) \neq \emptyset$.

More precisely, for some nontrivial invariant I:

$I \in \mathrm{Inv}(\Omega)$ and $I \in \mathrm{Inv}(\pi)$.

This is the source–gaze law.

4. Patterned Source, Random Gaze

If the source is strongly patterned and the gaze is weak, wandering, or partly random, discovery can still occur. The source may imprint itself on exploration.

$\mathrm{Inv}(\Omega) \neq \emptyset$, $\mathrm{Inv}(\pi)$ weak.

This is the regime of accident, play, computation, visual surprise, experimental mathematics, and unexpected analogy. The gaze does not create the structure; it stumbles into an invariant already available.

5. Random Source, Patterned Gaze

If the source has no stable invariant, but the gaze is patterned, then any apparent mathematics is receiver-induced.

$\mathrm{Inv}(\Omega) \approx \emptyset$, $\mathrm{Inv}(\pi) \neq \emptyset$.

This may produce useful formalism, but it is dangerous. It may be projection, numerology, overfitting, or artifact. Such mathematics requires severe certification against controls.

The admitted object is then not source-discovered but gaze-induced:

$[X]\pi$ is receiver-originated.

6. Random Source, Random Gaze

If both source and gaze lack invariant structure, no mathematics forms.

$\mathrm{Inv}(\Omega) \approx \emptyset$, $\mathrm{Inv}(\pi) \approx \emptyset$.

Then:

$\mathrm{Inv}(\Omega) \cap \mathrm{Inv}(\pi) = \emptyset$.

There is no theorem, no quotient, no object, no admissible identity.

Double randomness yields nothing.

7. Patterned Source, Patterned Gaze

If both source and gaze are patterned, discovery is still not guaranteed. Their invariants must overlap.

$\mathrm{Inv}(\Omega) \neq \emptyset$, $\mathrm{Inv}(\pi) \neq \emptyset$,

but discovery iff:

$\mathrm{Inv}(\Omega) \cap \mathrm{Inv}(\pi) \neq \emptyset$.

This explains why experts can miss new mathematics. Expertise is patterned, but often patterned for known invariants. A new field may require a gaze whose admissible invariants differ from inherited ones.

8. Theorem

Source–Gaze Discovery Theorem.

Let Ω be a mathematical presentation space and π an admissibility policy. A nontrivial mathematical object $[X]\pi$ forms from $X \in \Omega$ only if there exists an invariant I such that:

$I \in \mathrm{Inv}(\Omega) \cap \mathrm{Inv}(\pi)$.

If no such I exists, X may remain data, image, analogy, noise, or artifact, but it does not become mathematics under π.

Proof sketch. In Admissibility Algebra, objecthood is quotient formation under a policy. Quotient formation requires preserved identity. Preserved identity requires an invariant. If no invariant is both available from the source and admissible to the gaze, no equivalence judgment $X \equiv\pi Y$ can be certified. Therefore no quotient object $[X]\pi$ forms.

9. Certification

Invariant overlap is necessary but insufficient. A claimed invariant must survive certification.

$\mathrm{Cert}\pi(X)$ must show:

X satisfies the invariant,

the invariant survives admissible transformations,

controls fail,

artifacts are rejected,

the quotient is nontrivial,

the result is not merely imposed by π.

Thus discovery requires both overlap and discipline.

10. New Mathematics

New mathematics appears when π admits an invariant not already stabilized in the old quotient-space. Let Ω/π_old be the inherited object-space. A discovery occurs when X induces:

$[X]\pi\_new \notin \Omega/\pi\_old$,

while $Cert\pi\_new(X)$ remains valid.

The novelty is not raw difference. It is certified admissibility under a new or refined gaze.

11. Consequence for AI

An AI system searching for new mathematics should not merely generate formulas. It should search across:

presentation spaces Ω,

gazes π,

invariants Inv,

certificates Cert,

quotients Ω/π.

The deeper task is not expression generation. It is admissibility search.

A mathematical AI must learn not only what to output, but how to look.

12. Conclusion

New mathematics is born at the overlap of latent structure and admissible recognition. A patterned source can instruct a weak gaze. A patterned gaze can impose structure on weak sources, but risks hallucination. Double randomness produces no object. Double patterning succeeds only when invariants overlap.

The compact law is:

No invariant overlap, no mathematics.

The stronger law is:

No certified invariant overlap, no new mathematics.

13. References

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