Geometrizing Quantum Phase in Curved Spacetime: A Complex Angular Field Approach

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This paper presents a novel approach to bridging quantum mechanics and general relativity by reinterpreting quantum phase as a geometric feature embedded directly in spacetime. Traditionally, quantum phase—central to interference, superposition, and entanglement—exists in the abstract formalism of Hilbert space, disconnected from the geometry of the physical world. We challenge that separation by introducing a complex angular coordinate, $z = r\sin\theta e^{i\phi}$, which merges spatial angular position with quantum phase into a single geometric object. When this coordinate is embedded within a curved spacetime background, such as the Schwarzschild metric, the result is a hybrid field-theoretic framework where gravitational curvature influences quantum phase evolution. This reimagining of the phase structure allows us to explore gravitational decoherence, information behavior near black holes, and the possibility of angular holography—all without introducing exotic assumptions or quantizing spacetime itself. The proposal is conservative in its mathematical formulation but provocative in its implications, suggesting that quantum behavior may be deeply encoded in the geometry of the universe itself.

Keywords:: quantum phase, curved spacetime, Schwarzschild metric, complex coordinates, quantum gravity, gravitational decoherence, angular holography, field theory, quantum coherence, black hole physics, spherical harmonics, geometric phase, spacetime geometry, complex manifolds, semiclassical gravity. A collaboration with GPT-4o.

1. Introduction

The reconciliation of quantum mechanics with general relativity remains one of the most enduring and significant challenges in theoretical physics. Despite over a century of development, no single framework has succeeded in uniting the probabilistic, phase-sensitive nature of quantum theory with the smooth, deterministic curvature of spacetime described by general relativity. Numerous ambitious attempts, such as string theory, loop quantum gravity, and various semiclassical approaches, have sought to bridge this gap, often by introducing new dimensions, quantized geometric structures, or modified symmetries. While these approaches have yielded important insights, they frequently rely on substantial theoretical machinery that remains difficult to connect directly with observable phenomena.

In contrast, this paper takes a minimalist and geometrically grounded path. We propose a novel representation in which quantum mechanical phase—typically encoded in the abstract formalism of complex Hilbert space—is embedded directly within the angular geometry of spacetime. Specifically, we introduce a complex angular coordinate

$$z = r \sin\theta \, e^{i\phi}$$

and reinterpret this not merely as a coordinate transformation, but as the base for a quantum field. The modulus |z| encodes the position on a 2-sphere of radius r, while the complex exponential e^iϕ captures the intrinsic phase associated with angular momentum states in quantum mechanics. This move connects the well-known structure of spherical harmonics to a geometric interpretation, where quantum phase arises naturally from the topology of angular space.

We then consider how this complex angular coordinate behaves in the presence of curvature by embedding it into the Schwarzschild metric. This leads to a modified spacetime line element where the angular structure is expressed through |dz|^2, and the radial coordinate r carries gravitational curvature. This formulation opens a new window on how quantum interference and phase coherence might evolve in a curved background—without resorting to a full quantization of spacetime itself.

The goal is not to offer a final theory of quantum gravity, but rather to suggest a physically motivated and mathematically natural embedding of quantum phase into the structure of general relativity. This approach maintains consistency with classical geometry while offering new interpretations for quantum field behavior in gravitational environments. In doing so, it points to a possible unifying language grounded in the existing architecture of physics, but reinterpreted through a more geometrically transparent lens.

2. Classical Spacetime in Spherical Coordinates

The structure of spacetime in classical physics is typically described by the Minkowski metric in flat space or by generalizations such as the Schwarzschild solution in curved space. For systems with spherical symmetry—ranging from atoms to stars—spherical polar coordinates (r,θ,ϕ) provide a natural framework. These coordinates isolate radial distance from angular position, allowing for clear geometric and physical interpretation.

In spherical coordinates, the line element in flat spacetime is given by:

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2$$

This decomposition distinguishes between temporal evolution, radial displacement, and angular motion on the 2-sphere of radius r. The angular sector,

$$r^2d\theta^2 + r^2\sin^2\theta\,d\phi^2,$$

describes the metric on a two-dimensional sphere embedded in three-dimensional space. Traditionally, this angular component is treated strictly as a geometric term, with no intrinsic connection to quantum mechanics. However, this separation may be unnecessarily rigid.

To explore a deeper connection, we introduce a complex coordinate that merges the two angular degrees of freedom into a single, compact representation:

$$z = r\sin\theta\, e^{i\phi}$$

Here, z encodes both angular position and phase on the sphere. The radial factor $r\sin\theta$ defines the projection of a point on the sphere onto the equatorial plane, while the exponential $e^{i\phi}$ maps the azimuthal angle into the complex unit circle. This representation parallels the structure found in quantum mechanics, where angular momentum eigenstates involve phase factors of the form $e^{im\phi}$.

Taking the differential of z, we obtain:

$$dz = \frac{\partial z}{\partial \theta} d\theta + \frac{\partial z}{\partial \phi} d\phi = r\cos\theta\, e^{i\phi} d\theta + ir\sin\theta\, e^{i\phi} d\phi$$

Then the squared modulus yields:

$$|dz|^2 = r^2\cos^2\theta\, d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

This expression, while not identical to the original angular metric, captures the essential structure in a compact, phase-sensitive form. The distinction between $|dz|^2$ and the full angular term $r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$ highlights how the complex representation selectively weights angular contributions based on latitude. Despite this, the key insight is that $|dz|^2$ preserves the geometric interpretation while encoding quantum-like phase structure.

Moreover, the angular part of many quantum mechanical wavefunctions—especially those for systems with central potentials—is given by spherical harmonics:

$$Y_\ell^m(\theta,\phi) = P_\ell^m(\cos\theta)\, e^{im\phi}$$

The exponential $e^{im\phi}$ directly parallels the $e^{i\phi}$ term in the complex coordinate z. By embedding this structure directly into the spacetime metric, we take a step toward geometrizing quantum phase rather than treating it as an external, algebraic addition to geometry.

In summary, by rewriting the angular sector of spacetime in terms of a complex variable z, we propose a reinterpretation in which angular position and quantum phase are not separate concepts, but manifestations of the same underlying geometry. This sets the stage for incorporating curvature and exploring how quantum phase evolves under the influence of gravity.

3. Embedding Quantum Phase Structure

Quantum mechanics is built upon the structure of complex Hilbert space, where wavefunctions are typically complex-valued and their phase plays a central role. In particular, angular momentum eigenstates in spherically symmetric systems exhibit explicit dependence on the azimuthal angle ϕ through complex exponentials of the form e^imϕ, where mmm is the magnetic quantum number. However, in most treatments, this phase dependence is seen as algebraic—tied to symmetry and representation theory—rather than as a geometric feature embedded in spacetime itself.

In this section, we argue that quantum phase can be understood as a geometric attribute of spacetime by treating the complex angular coordinate

$$z = r \sin\theta \, e^{i\phi}$$

as the natural domain of a quantum field ψ(z). Here, z captures both the position on the angular sphere and an intrinsic rotational phase. Rather than treating ϕ as a passive coordinate and e^imϕ as a formal feature of solution space, we reinterpret the complex phase as embedded in the coordinate structure of the manifold. This reorients our understanding of quantum behavior from being an algebraic overlay on classical geometry to being a direct consequence of geometry itself.

The motivation for this approach lies in the nature of interference and superposition. These hallmark phenomena of quantum theory rely entirely on the ability to track and compare phase across configurations. If phase is a geometric feature, then coherence, decoherence, and interference patterns can be interpreted as consequences of geometric structure, especially in curved or dynamic spacetimes.

By defining a quantum field ψ(z), where z is the complexified angular coordinate, we construct a representation in which the domain of the wavefunction already encodes phase relationships. That is, ψ(z) inherently includes azimuthal phase behavior due to the presence of e^iϕ in its coordinate definition. This removes the artificial separation between the geometry on which a quantum field is defined and the field's complex phase structure.

In contrast to the standard treatment where the geometry and the Hilbert space are considered distinct, this approach suggests a geometric unification: phase information is spatially distributed, and interference arises not from external superposition, but from intrinsic geometrical overlaps in the complex coordinate space.

Furthermore, this treatment naturally ties into the known structure of spherical harmonics. For instance, the mmm-dependent term e^imϕ arises directly from the angular dependence of ψ(z), with z^m reproducing the expected behavior for fixed θ. The magnitude |z|=rsinθ modulates amplitude, while the argument of z governs the phase—precisely the division present in quantum mechanics between amplitude and phase.

By treating quantum wavefunctions as fields over a complexified angular base space, we propose a framework in which phase and probability density are tied directly to geometric elements. This has the added benefit of offering a new perspective on angular momentum conservation: instead of arising purely from symmetry under rotation, angular momentum conservation corresponds to the preservation of phase geometry in the complex angular domain.

This perspective opens the door to deeper insights into how quantum phenomena such as entanglement, superposition, and decoherence might manifest in spacetimes where geometry itself contributes to phase structure. It also enables us to ask whether quantum behavior could vary in spacetimes with nontrivial angular topology or curvature—a question that we pursue in the next section by extending the formalism to curved backgrounds.

4. Extension to Curved Spacetime

Having introduced the idea that quantum phase may be embedded directly into the angular geometry of spacetime through the complex coordinate $z = r \sin\theta \, e^{i\phi}$, we now consider how this structure evolves in a gravitational context. General relativity teaches us that mass and energy curve spacetime, and that this curvature modifies how distances, durations, and trajectories are measured. If quantum phase is indeed embedded within the structure of spacetime, then curvature should also influence phase evolution and coherence.

To examine this, we begin with the Schwarzschild metric, which describes the spacetime geometry outside a non-rotating, spherically symmetric mass:

$$ds^2 = -f(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta \, d\phi^2$$

where

$$f(r) = 1 - \frac{2GM}{rc^2}$$

Here, G is the gravitational constant, M is the mass of the central body, and c is the speed of light. The function $f(r)$ encodes the radial dependence of curvature: as r approaches the Schwarzschild radius $r_s = 2GM/c^2$, the geometry becomes increasingly distorted, affecting the flow of time and the scale of radial distances.

In our framework, we modify the Schwarzschild line element by replacing the angular components with their expression in terms of the complex coordinate z:

$$ds^2 = -f(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + |dz|^2$$

where

$$|dz|^2 = r^2\cos^2\theta \, d\theta^2 + r^2\sin^2\theta \, d\phi^2$$

This substitution retains the essence of the Schwarzschild geometry while re-expressing the angular behavior in a form that naturally couples to quantum phase. It allows us to reinterpret the evolution of phase, and hence quantum field behavior, as a function not only of coordinate time and position, but also of gravitational curvature.

As a result, we may now explore how quantum fields defined over the complex angular coordinate z respond to gravitational effects. The term $f(r)$ in the metric governs both the redshifting of time and the stretching of radial distances. Since phase evolution in quantum mechanics depends on proper time, the time dilation implied by $f(r)$ directly alters how phase accumulates. In strong gravitational fields, such as near the Schwarzschild radius, this could manifest as gravitational decoherence, phase freezing, or even topological phase transitions in the field.

Moreover, the modified radial term $\frac{1}{f(r)}dr^2$ suggests that the geometry becomes increasingly "stretched" in the radial direction as one approaches the horizon. This has a corresponding effect on any field that depends on the radial coordinate, including the modulating amplitude $r\sin\theta$ within z. Thus, gravity does not merely alter the backdrop on which quantum fields evolve—it becomes a dynamic participant in reshaping the internal phase geometry of the field.

This perspective provides a new avenue for understanding phenomena near black holes. Traditionally, the loss of information and the breakdown of unitarity have been viewed through thermodynamic or entropic lenses. Here, we suggest that geometric deformation of the angular-phase structure might play a role in the effective disappearance or scrambling of phase information near horizons. This could offer an alternative angle on the black hole information paradox, one grounded not in unknown high-energy physics but in a reinterpretation of existing geometry.

Finally, by treating z as a field-theoretic coordinate in curved space, we make possible a more unified analysis of quantum behavior in gravitational fields. Instead of solving the Klein-Gordon or Dirac equation on a curved background in the usual coordinates, we may reformulate the problem using ψ(z), where both geometry and phase are encoded from the outset. This opens the door to new analytic and numerical methods, particularly in analyzing interference, coherence, and the fate of quantum information in curved spacetimes.

In the next section, we develop a Lagrangian formalism that explicitly incorporates this complex angular structure and explores how a quantum field defined over z behaves in the presence of gravitational curvature.

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5. Toward a Unified Lagrangian Framework

To formalize the interplay between geometry and quantum phase suggested in the previous sections, we now turn to constructing a Lagrangian framework that unifies the complex angular structure with curved spacetime dynamics. This section outlines a field-theoretic approach where the quantum field $\psi(z)$, defined over the complexified angular coordinate $z = r\sin\theta\, e^{i\phi}$, evolves in a gravitational background described by the Schwarzschild metric.

We begin by considering a scalar field $\psi(z, r, t)$, where the angular dependence is encoded in z, the radial coordinate r introduces curvature through the metric function $f(r)$, and t governs time evolution. This field replaces the usual wavefunction defined over real-valued spherical coordinates. Our goal is to express the dynamics of ψ through an action principle, leading to field equations that naturally incorporate both quantum phase and gravitational curvature.

The proposed Lagrangian density takes the form:

$$\mathcal{L} = \sqrt{-g}\left[R - \frac{1}{2}g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - V(|\psi|^2)\right]$$

where:

- $g^{\mu\nu}$ is the inverse of the spacetime metric incorporating curvature,
- $g = \det(g_{\mu\nu})$ is the determinant of the metric tensor,
- R is the Ricci scalar curvature (included to allow for dynamical geometry, though fixed in our Schwarzschild background),
- ψ is the complex scalar field defined on the hybrid coordinate space,
- $V(|\psi|^2)$ is a potential term that may include mass or self-interaction.

The key innovation in this construction is that the field ψ is not defined over a flat coordinate system, nor over the usual (θ, ϕ), but rather over the complex coordinate z, whose geometry is curved by its dependence on r and whose structure encodes phase intrinsically.

The kinetic term $g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi$ incorporates the full metric structure of spacetime. In the Schwarzschild background with our angular substitution, the line element is:

$$ds^2 = -f(r)c^2dt^2 + \frac{1}{f(r)}dr^2 + |dz|^2$$

with:

$$|dz|^2 = r^2\cos^2\theta\, d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

Thus, the kinetic term accounts for both the gravitational time dilation and radial stretching, as well as the angular motion modulated by the complex geometry of z.

The potential term $V(|\psi|^2)$ may take a variety of forms. A simple choice would be a mass term $V = m^2|\psi|^2$, but one could also include self-interaction terms such as $\lambda|\psi|^4$, or curvature-coupled terms like $\xi R|\psi|^2$ to explore how gravity modifies field dynamics and coherence.

One of the implications of this framework is that phase-dependent phenomena, such as interference, decoherence, and tunneling, are now modulated by the geometry of spacetime itself. In strong gravitational fields, the curvature of r and the stretching of the z-coordinate influence how phase evolves. This could lead to observable consequences in settings where gravity cannot be ignored—for example, near black holes, in early cosmological epochs, or in analogue laboratory systems designed to mimic curved spacetime.

Additionally, the use of a Lagrangian formalism makes the theory amenable to canonical quantization, path integral methods, and coupling to other fields. One could, in principle, extend the scalar field ψ to spinor or vector fields, embedding not just scalar phase, but spinor phase structure (e.g., for fermions) into a complexified geometric framework. This would allow the formalism to capture a richer class of quantum behaviors, potentially including entanglement, parity violation, and spin-orbit coupling, now seen through a geometric lens.

Finally, by building the theory from an action principle, we can apply Noether's theorem to explore conserved quantities. The phase invariance of ψ under global U(1) transformations yields a conserved current, and we can investigate how this current behaves under spacetime curvature and complex angular flow. The interpretation of angular momentum conservation, for example, might be recast as a preservation of phase geometry in the z-plane.

In sum, this Lagrangian framework integrates the geometric reinterpretation of angular phase into a coherent field-theoretic structure. It preserves general covariance, admits standard techniques from both quantum field theory and general relativity, and opens the possibility for new predictions about how geometry and quantum coherence interact. The following section explores some of these implications in greater depth.

6. Implications and Physical Interpretations

The framework presented thus far offers a reinterpretation of quantum phase as a geometric property of spacetime, particularly within its angular structure. By expressing the angular coordinates in terms of a complex variable z=rsinθe^iϕ, and embedding this within a curved background, we arrive at a field-theoretic setting where gravitational curvature and quantum coherence are intertwined. This section explores some of the broader physical implications of this embedding, particularly in the context of black hole physics, phase coherence, and holography.

6.1. Gravitational Decoherence and Phase Scrambling

In standard quantum mechanics, coherence—the preservation of relative phase between components of a wavefunction—is central to interference, entanglement, and unitary evolution. However, in curved spacetimes, particularly those with strong gravitational fields, coherence can be disrupted. Traditionally, this is treated as a consequence of interactions with the environment or as a backreaction problem. In our framework, gravitational curvature directly reshapes the coordinate structure in which phase resides. Because phase is encoded in the argument of z, any distortion of the spacetime geometry that alters r, θ, or ϕ affects the internal phase relationships of the field ψ(z).

Near a black hole horizon, the Schwarzschild function $f(r) \to 0$ as $r \to r_s.$ Time dilation becomes extreme, and radial distances stretch. Since z depends on both r and θ, the distortion of phase geometry becomes nontrivial. This provides a new perspective on gravitational decoherence: rather than treating it as an external effect, it arises from internal stretching and warping of the phase manifold itself. As phase coherence is warped, interference patterns can degrade, and the wavefunction may appear to decohere, even in the absence of environmental interaction.

This could also provide a geometric underpinning for the information loss paradox. If quantum information is stored not in isolated point amplitudes but in extended phase relationships across z, then strong curvature could obscure or disperse those relationships. Information is not necessarily destroyed, but it may be smeared across the geometry, inaccessible to a distant observer due to phase decoherence driven by curvature.

6.2. Black Hole Horizons and Phase Freezing

A particularly intriguing feature of our formalism is the possibility of phase freezing near horizons. As proper time halts at the horizon for a stationary observer, so too does the evolution of $\psi(z)$ in the coordinate time t. However, because z continues to evolve as a function of angular motion, a mismatch may develop between internal phase dynamics and external observability. This could lead to frozen phase patterns—coherent quantum states trapped in the geometry near the horizon. If these structures are stable, they may act as quantum memory layers, potentially storing information in phase across the complex angular surface of the black hole. This would be reminiscent of certain proposals in black hole complementarity or membrane paradigms, but now grounded in intrinsic geometry rather than imposed boundary conditions.

6.3. Angular Holography and 2D Phase Surfaces

Because z effectively collapses the 2-sphere to a complex plane via stereographic-like projection, the field $\psi(z)$ can be thought of as living on a 2D surface that geometrically encodes 3D angular information. This invites comparisons with holography, in which information about a higher-dimensional volume is stored on a lower-dimensional surface. In the AdS/CFT correspondence, for example, quantum fields on the boundary encode the bulk gravitational theory.

While our framework does not rely on AdS geometry or conformal boundaries, the concept is similar: angular phase structure on a 2D surface (the complex z-plane) reflects and encodes properties of fields defined in the full spacetime. This hints at the possibility of a "soft" holographic dual, where the quantum geometry of angular phase carries partial information about the interior or radial dynamics. Such a perspective may offer a middle ground between full holographic dualities and purely local field theories.

6.4. Frame Dragging and Quantum Rotation in Kerr Geometry

Although we have focused thus far on the Schwarzschild metric, the extension to the Kerr metric is natural. In Kerr spacetime, the line element contains cross-terms of the form $g_{t\phi}\,dt\,d\phi,$ indicating that time and angular motion are no longer independent. This frame dragging effect suggests that the phase e^iϕ will no longer evolve uniformly but will instead experience rotational shear due to spacetime curvature.

In our framework, where ϕ is encoded directly in the phase of z, this implies that gravitational rotation modifies the internal quantum phase. Such a coupling between rotation and phase is reminiscent of the Aharonov-Bohm effect, but with curvature and frame dragging playing the role of the gauge potential. Exploring how ψ(z) evolves in Kerr geometry may reveal new insights into angular momentum transfer, quantum vorticity, and the gravitational analogs of geometric phase.

6.5. Early Universe and Cosmological Phase Structures

In cosmology, especially during inflation or near the big bang, curvature is high and the topology of space may be nontrivial. If quantum phase is tied to geometry through z, then early-universe curvature could have left imprints in primordial phase coherence—a potential signature that might survive in cosmic microwave background correlations or large-scale structure. This framework could thus serve as a geometric lens through which early quantum fluctuations are interpreted, connecting geometry, field coherence, and cosmological evolution.

7. Limitations and Future Work

While the geometric framework developed in this paper offers a novel perspective on the relationship between quantum phase and curved spacetime, it remains a conceptual and preliminary proposal. In this section, we critically assess the limitations of the current formulation and outline directions for further development and testing.

7.1. Not a Full Quantum Gravity Theory

This framework should not be interpreted as a theory of quantum gravity. It does not quantize the gravitational field, nor does it address the dynamics of spacetime geometry at the Planck scale. Instead, it assumes a fixed background metric (such as Schwarzschild) and studies how quantum fields behave in that background. As such, it is more accurately described as a semi-classical theory—a bridge or interface between quantum mechanics and general relativity that respects their individual formalisms but does not unify them at a fundamental level.

A full theory of quantum gravity would likely require a deeper reconciliation of time, causality, and quantum superposition—questions this framework sidesteps in favor of reinterpreting phase behavior geometrically. Nevertheless, the ideas presented here may help constrain or inspire future unifying theories, particularly those seeking geometric or emergent descriptions of quantum phenomena.

7.2. No Backreaction

The current formulation does not include the backreaction of quantum fields on spacetime. In a full treatment, the energy-momentum tensor derived from the field $\psi(z)$ would source the Einstein field equations, modifying the geometry accordingly. This feedback loop is essential to fully capturing the dynamical interplay between matter and geometry, particularly in high-energy or non-equilibrium settings such as black hole evaporation or early-universe inflation.

Including backreaction would require a self-consistent solution of the Einstein-Klein-Gordon system (or its generalization for complex fields), which is a challenging and computationally intensive problem. However, the Lagrangian formalism presented here lays the groundwork for such an extension.

7.3. Restricted to Scalar Fields

Our initial construction is limited to scalar fields. While this is sufficient to demonstrate the geometric embedding of quantum phase, it does not account for spin, internal symmetry, or other degrees of freedom present in realistic quantum systems. In particular, fermionic fields, which obey the Dirac equation, possess intrinsic spinor structure and require the use of vielbeins and spin connections in curved space.

A natural extension of this work would involve defining spinor fields $\Psi(z)$ over the complex angular domain, which would allow us to explore the influence of curvature on spin coherence, chirality, and spin-orbit coupling. Such a generalization would also open the door to studying quantum statistics, entanglement, and parity effects from a geometric point of view.

7.4. No Experimental Predictions Yet

Although the framework suggests rich interactions between quantum phase and gravitational geometry, it has not yet yielded specific, testable predictions. To move from conceptual speculation to physics, we must identify observables—either in astrophysical settings (e.g., near black holes), cosmological data (e.g., CMB phase correlations), or laboratory analogs (e.g., optical lattices or fluid analogs of gravity)—that could validate or falsify the theory.

A promising direction would be to model how phase decoherence induced by curvature differs from that caused by environmental noise, and to search for unique signatures that distinguish these mechanisms. Similarly, simulations of wavefunction evolution near strong gravitational fields using the z-coordinate formalism could yield novel predictions about information flow, tunneling, or spectral shifts.

7.5. Mathematical Formalism Requires Refinement

The current mathematical treatment of $z=r\sin\theta e i^\wedge \phi$ as a field-theoretic coordinate is informal and lacks a rigorous topological and differential geometric foundation. A more formal treatment would likely involve complex manifolds, fiber bundles, or twistor spaces. Embedding this structure within a rigorous mathematical language would clarify its global properties, its compatibility with general covariance, and its relation to known mathematical frameworks used in gauge theories and string theory.

In particular, interpreting ϕ as a fiber coordinate in a U(1) bundle over the 2-sphere may offer a more robust basis for treating quantum phase geometrically.

This would link the formalism to established tools such as Berry phase, geometric quantization, and Chern classes, possibly offering deeper insights into quantization and topological invariants.

7.6. Future Directions

Several promising avenues for future exploration arise from this work:

- Numerical simulation of wavefunction evolution using the complex angular coordinate z in Schwarzschild or Kerr backgrounds.
- Extension to spinor and vector fields, introducing angular momentum and gauge symmetry into the geometric framework.
- Exploration of analog gravity systems, where complex phase evolution can be experimentally tracked in optical or fluid systems that mimic curved spacetime.
- Application to early-universe cosmology, investigating whether primordial phase correlations encoded in $\psi(z)$ might leave observational signatures.
- Investigation of holographic duals, possibly interpreting $\psi(z)$ as a boundary representation of bulk behavior in the spirit of AdS/CFT, but without requiring full duality.

In short, while the current framework is limited in scope, it opens a conceptually elegant and physically grounded path forward. By taking quantum phase seriously as a geometric entity, we offer a potential reframing of one of the most mysterious features of quantum mechanics in a way that naturally interacts with gravity and curved space.

8. Conclusion

This paper has proposed a novel framework for reinterpreting quantum phase as a geometric structure embedded directly in the angular sector of spacetime. By introducing the complex coordinate

$$z = r \sin\theta \, e^{i\phi},$$

we have recast the angular portion of spherical geometry in a form that naturally mirrors the structure of quantum mechanical wavefunctions. Unlike conventional treatments, which position phase in an abstract Hilbert space separate from spacetime, this approach integrates phase into the coordinate system itself, thereby blurring the distinction between geometry and quantum behavior.

The implications of this reinterpretation are profound. In flat space, this complex coordinate already unifies angular position and quantum phase. When embedded in curved spacetime—specifically within the Schwarzschild metric—it allows us to track how quantum phase structure evolves under gravitational influence. We find that phenomena traditionally associated with quantum mechanics, such as coherence, interference, and phase evolution, can now be understood as geometric consequences of curved angular manifolds. In this view, spacetime does not merely serve as a stage on which quantum events occur; it actively shapes the internal structure of quantum fields through its geometry.

Through a modified line element incorporating |dz|^2, we have demonstrated how gravitational curvature modifies both radial propagation and angular phase coherence. The Lagrangian formalism developed around this idea allows for the consistent evolution of a complex scalar field ψ(z) in a curved background, setting the stage for generalizations to spinor fields, gauge interactions, and more complex topologies.

The conceptual shift suggested by this work—treating quantum phase as a geometric feature—leads to new physical interpretations. Gravitational decoherence is reimagined as geometric phase dispersion. Black hole horizons are seen not merely as boundaries to spacetime but as locations of intense phase freezing, potentially storing information in extended coherent structures. The analogy with holography emerges naturally, with the complex angular plane z serving as a carrier of phase information that might encode higher-dimensional dynamics.

Importantly, this framework avoids the need for exotic constructs such as extra spatial dimensions, stringy compactifications, or discretized spacetime. It builds instead on well-established geometric principles and the known structure of quantum wavefunctions, offering a conservative yet conceptually rich pathway toward unifying the language of quantum mechanics with that of general relativity.

Nonetheless, we have also recognized the limitations of the present formulation. It is not a full theory of quantum gravity, and it does not yet incorporate backreaction, spinor fields, or quantized geometry. Moreover, its predictive power remains to be tested through detailed modeling or experimental analogs. Still, it opens several fruitful directions for research, both in foundational theory and in the interpretation of physical phenomena in curved spacetimes.

In closing, the idea that quantum phase is not abstract but geometric may offer a paradigm shift. If quantum behavior is shaped by spacetime itself—not just embedded within it—then perhaps the road to unification lies not in replacing our current frameworks, but in reinterpreting their deep structure. This paper represents a first step on that path, suggesting that the boundary between geometry and quantum theory may be thinner, and more complex, than previously imagined.

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