

Analog Algebra and the Future of AI: Embedding Abstract Structures in Continuous Computation

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The 2025 Chinese RRAM-based analog chip, capable of solving matrix equations with near-digital precision, signals a paradigm shift in computation. For the first time in decades, analog systems—once dismissed as obsolete—are reemerging as contenders in high-performance computing and artificial intelligence. Yet the implications reach far beyond matrix inversion or signal processing. Analog computation raises a profound theoretical question: *what kinds of mathematics can be realized physically?* This paper explores how the laws of abstract algebra—from groups and rings to categories and homotopies—can be constrained, embedded, and expressed through continuous analog dynamics. We examine how algebraic operations become geometric flows, how categorical composition manifests as signal propagation, and how homotopy paths can appear as convergence trajectories. When integrated into AI architectures, these analog embeddings could unite symbolic abstraction with physical energy minimization, transforming reasoning from a digital sequence of operations into a smooth process of alignment. The goal is not to replace digital AI, but to complement it with an analog foundation where computation, geometry, and cognition become indistinguishable.

Keywords: analog computation, RRAM, abstract algebra, category theory, homotopy type theory, analog AI, continuous computation, energetic convergence, matrix equation solving, Logistics, $q \rightarrow \tau$ alignment.

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Outline:

1. Introduction: Analog Revival and the 2025 RRAM Breakthrough
 - Overview of the Chinese analog chip
 - Relevance to the resurgence of physical computation
 - Why analog matters for future AI paradigms
2. Theoretical Divide: Discrete Algebra vs. Continuous Physics
 - Nature of symbolic vs. analog reasoning
 - Mapping algebraic operations to smooth manifolds
 - Precision and tolerance as new logical boundaries
3. Embedding Abstract Algebra in Analog Systems
 - Representing groups, rings, and fields as continuous transformations
 - From operations to flows: differential realization of algebraic laws
 - Fixed points and convergence as algebraic truth
4. Categories and Concrete Analogues
 - Restricting to concrete categories of continuous maps
 - Composition as physical wiring and cascaded flow
 - Analog categories and signal-processing metaphors
5. Homotopy Type Theory in the Analog Domain
 - Paths as continuous proofs
 - Energetic convergence as constructive homotopy
 - The logic of approximation vs. symbolic equality
6. Analog Algebra in Artificial Intelligence
 - Hybrid symbolic-continuous architectures
 - Differential categorical AI and energetic alignment
 - Mapping $q(t) \rightarrow \tau(t)$ as a physical realization of learning

7. Mathematical Criteria for Analog Computability

- Embedding, closure, and convergence conditions
- Precision bounds and physical computability limits

8. Logistics and the Ethics of Alignment

- Energy, truth, and the metaphysics of convergence
- Analog computation as a moral model of resonance

9. Conclusion: From Symbol to Substance

- Algebra as living geometry
- The next frontier of AI: computation as embodiment

References

1. Introduction: Analog Revival and the 2025 RRAM Breakthrough

The year 2025 marked a turning point in the trajectory of computation. A team at Peking University unveiled an analog resistive-random-access-memory (RRAM) chip capable of solving matrix equations $Ax = b$ with a precision once thought exclusive to digital processors. Reported in *Nature Electronics*, the chip achieved 24-bit fixed-point accuracy through a hybrid method combining low-precision analog inversion with high-precision digital refinement. Its physical substrate—a 1T1R RRAM crossbar array—embodied the mathematical matrix itself: voltages represented vector inputs, conductances encoded matrix coefficients, and resulting currents directly yielded the solution vector. Benchmarks indicated up to 1000× higher throughput and 100× energy efficiency compared to state-of-the-art GPUs. In one stroke, the demonstration revalidated the central intuition of analog computation: that physics itself can *solve equations* without simulating them.

This breakthrough reignited interest in physical computation—the notion that computation is not limited to symbolic manipulation but can arise from the lawful behavior of matter. From the 1940s differential analyzers of Vannevar Bush to the memristive and photonic systems of today, each era has rediscovered that continuous systems perform inherently parallel and energy-minimizing operations. In the context of artificial intelligence, where vast neural architectures are constrained by energy and latency, such analog processes offer not merely efficiency but *ontological insight*: intelligence may ultimately emerge from *continuous alignment* rather than discrete calculation. Analog computing thus reopens a frontier that merges mathematics, physics, and cognition—an exploration of how abstract structures might live and compute within the real world itself.

2. Theoretical Divide: Discrete Algebra vs. Continuous Physics

At the heart of computation lies a philosophical divide between symbolic abstraction and physical embodiment. Digital logic, and by extension abstract algebra, operates through discrete reasoning—the manipulation of symbols according to exact axioms. A bit is either 0 or 1, a group operation either satisfies closure or it does not. Truth is binary and absolute, grounded in logical consistency rather than energetic cost. Analog systems, by contrast, inhabit a continuous world: their states evolve over real-valued manifolds, governed by smooth dynamics and feedback. A voltage can represent any value within a continuum, and the system’s “computation” unfolds through physical equilibrium, not through sequential rule application. The analog machine does not *calculate* in the digital sense—it *settles*.

This distinction translates mathematically into two complementary paradigms:

- Symbolic reasoning represents operations as mappings between discrete elements of algebraic structures—sets, groups, or rings.
- Analog reasoning interprets those same operations as flows on smooth manifolds, where computation corresponds to the evolution of trajectories toward stable attractors that satisfy the governing equations.

To bridge these realms, one introduces a representational embedding $\varphi: A \rightarrow M$, mapping algebraic elements $a_i \in A$ into points on a manifold M . Each algebraic operation $o: A^n \rightarrow A$ is then approximated by a smooth function $f: M^n \rightarrow M$, satisfying

$$\|\varphi(o(a_1, \dots, a_n)) - f(\varphi(a_1), \dots, \varphi(a_n))\| \leq \varepsilon,$$

where ε denotes the allowable deviation or *tolerance*. In this framework, precision and tolerance replace truth and falsity as the operative logical boundary. An analog computation need not yield an exact algebraic identity; it need only fall within a small region of convergence.

This shift introduces a new epistemic logic—one not of certainty, but of continuous approximation. The axioms of algebra become attractor conditions, and proofs become trajectories of convergence. Closure and associativity are replaced by energetic stability and dynamical consistency. Thus, while symbolic algebra idealizes computation as manipulation of immutable truths, analog computation reframes it as the *approach* to truth within the limits imposed by matter. Between the discrete algebra of the mind and the continuous algebra of nature lies an emerging synthesis: the mathematics of alignment, where meaning itself is defined by the rate and fidelity of convergence.

3. Embedding Abstract Algebra in Analog Systems

The embedding of abstract algebra into analog computation begins with a recognition: algebraic structures, though discrete by definition, can be realized as continuous transformations when mapped into the geometry of physical systems. A group operation, a ring multiplication, or a field addition can all be expressed as flows or transformations on a manifold where the laws of algebra appear as constraints on dynamics. Analog systems thus act as *geometric interpreters* of algebraic law, replacing symbolic manipulation with continuous evolution toward equilibrium.

3.1 Representing Groups, Rings, and Fields as Continuous Transformations

Consider a group $(G, \cdot)$. In symbolic mathematics, the group law is an abstract binary operation satisfying closure, associativity, identity, and inverses. In an analog context, one can represent

G as a Lie group, where elements correspond to points on a smooth manifold, and the operation $g \cdot h$ is a differentiable map

$$\Phi: G \times G \rightarrow G.$$

Here, composition becomes a physical transformation—such as rotation, translation, or phase shift—within a continuous space. A field $(F, +, \times)$ can likewise be embedded in electrical or optical domains: voltages add linearly, and multiplicative scaling emerges through operational amplifiers, transconductance devices, or photonic interference patterns. Rings, which unite additive and multiplicative structures, find natural analogs in coupled oscillators or wave interference systems, where superposition (addition) and modulation (multiplication) coexist within the same physical substrate.

In each case, the algebraic element is reinterpreted as a state vector, and the algebraic operation becomes a smooth transformation acting on that state. This allows abstract algebra to be enacted rather than symbolically executed—a law of motion rather than a rule of syntax.

3.2 From Operations to Flows: Differential Realization of Algebraic Laws

To compute analogically is not merely to map algebraic laws but to allow them to unfold dynamically. An equation such as $f(x) = 0$ can be turned into a *flow*

$$\frac{dx}{dt} = -\nabla |f(x)|^2,$$

whose steady states correspond to the algebraic solutions. In this sense, analog computation transforms algebraic identity into differential stability: the condition of truth becomes an attractor in phase space.

For example, matrix inversion—a linear algebraic operation—can be realized as a current flow in an RRAM array where Kirchhoff’s and Ohm’s laws inherently satisfy the linear relations. Similarly, polynomial roots can be found by setting up oscillator networks whose coupling potentials encode the coefficients; their equilibrium amplitudes represent the roots. Even non-linear group operations, such as those on manifolds with curvature, can be implemented through feedback circuits that conserve an invariant energy functional, ensuring associativity “up to convergence.”

3.3 Fixed Points and Convergence as Algebraic Truth

In symbolic algebra, truth is absolute: either an equation holds or it does not. In analog computation, truth emerges as convergence—the stable configuration toward which the system

relaxes. The fixed point replaces the proof as the measure of validity. When a differential system reaches a state where its internal forces balance, the corresponding algebraic relation is effectively satisfied.

Let $F(x)$ denote the vector field governing an analog system. The fixed points x^* satisfying $F(x^*) = 0$ correspond to algebraic truths encoded in physical law. A stable fixed point indicates a robust truth; an unstable one corresponds to a fragile or degenerate algebraic condition.

This framework redefines algebraic reasoning in physical terms: closure corresponds to conservation of energy, associativity to commutativity of flows, and identity to equilibrium. Algebraic truth thus becomes *an emergent property of stability*.

By embedding abstract algebra into analog systems, mathematics crosses the boundary from formal description to ontological participation. Equations no longer describe the world—they *become* it. The flow of current, the oscillation of a field, or the damping of an error signal stands in for the algebraic relation itself. The analog machine is not computing symbols that *represent* algebra—it is physically *being* the algebra it computes.

4. Categories and Concrete Analogues

Category theory, often called “the mathematics of mathematics,” provides the most abstract language available for describing structure and transformation. In categorical terms, a category consists of *objects* and *morphisms* (arrows) between them, together with rules for *composition* and *identity*. These simple notions can describe everything from logic to geometry to computation. Yet, for analog realization, one must narrow the scope from symbolic universality to concrete categories, where both objects and morphisms possess continuous, physical interpretation. In such a setting, composition is not symbolic concatenation but literal signal propagation—the chaining of physical transformations in time and space.

4.1 Restricting to Concrete Categories of Continuous Maps

In a fully abstract category, objects may be anything—sets, types, topoi—and morphisms can be mere logical correspondences. Analog hardware, however, cannot instantiate purely symbolic relations. It can only embody concrete categories, where each object corresponds to a manifold (e.g., a vector space, a voltage field, an optical mode) and each morphism is a continuous map or transformation between manifolds.

Formally, an analog category $\mathcal{C}_a$ can be described as:

- Objects: spaces of continuous signals or state vectors $X, Y, Z \subset \mathbb{R}^n$.
- Morphisms: smooth maps $f: X \rightarrow Y$ realized by physical transformations (resistive networks, amplifiers, oscillators).
- Composition: function composition $g \circ f$ corresponding to the physical chaining of transformations.

Here, every arrow in the category is backed by a measurable transfer function, and every commutative diagram corresponds to a physically testable flow network. Commutativity no longer means logical equality—it means *signal coherence*.

4.2 Composition as Physical Wiring and Cascaded Flow

Composition, the defining operation of category theory, finds a natural analog in physical wiring. When the output of one circuit is connected to the input of another, the composite system realizes the mathematical composition $g \circ f$. Cascaded transformations form a signal pipeline, where each morphism processes, transforms, or filters information carried by a continuous medium.

In this way, analog circuits inherently obey categorical logic:

- The identity morphism corresponds to a direct connection—no transformation, no delay.
- Associativity emerges because physical connections can be rearranged without altering end-to-end behavior (up to impedance and timing tolerances).
- Functoriality appears when one layer of transformation (such as amplification or modulation) consistently maps between categories of physical states, preserving their compositional structure.

Composition in analog computation thus replaces logical concatenation with *energetic continuity*. A properly designed analog cascade is not a sequence of isolated computations—it is a single, distributed process in which energy and information flow seamlessly through a hierarchy of morphisms.

4.3 Analog Categories and Signal-Processing Metaphors

Within this framework, signal processing becomes the natural metaphor for categorical analog computation. Each stage of a signal pathway represents a morphism; each node, an object; and each closed feedback loop, a self-morphism capturing a recursive or self-referential process. A complete analog machine can be viewed as a commutative diagram in motion—currents and voltages flowing along morphisms that must balance, synchronize, and stabilize.

For instance, in an analog neural network, weighted summation corresponds to one morphism, non-linear activation to another, and the feedback connections form higher-order compositions. If we interpret this structure categorically, such a network is not a collection of discrete operations but a continuous functor mapping input manifolds to output manifolds through smooth transformations. Analog composition thereby manifests as functional integration, where temporal and spatial continuity replaces stepwise computation.

This interpretation situates analog computation within a differentiable category—a realm where morphisms are differentiable maps and composition corresponds to chained derivatives or transformations. The very process of backpropagation in AI can be viewed as navigating such a category, propagating adjustments along the same morphic pathways that carry signals forward.

Ultimately, the analog category offers a tangible vision of categorical reasoning: structure emerges not from syntactic relationships but from energetic coherence. Morphisms are realized as flows of matter or energy, and composition is the choreography of these flows within a manifold of continuity. This reframing turns category theory from a static grammar of mathematics into a living geometry of computation, where the world itself behaves as a commutative diagram written in voltages, frequencies, and light.

5. Homotopy Type Theory in the Analog Domain

Homotopy Type Theory (HoTT) unites logic, topology, and computation by replacing equality with paths of equivalence—continuous transformations that witness how one structure can deform into another. In classical logic, equality is static and absolute: two terms are either identical or distinct. In HoTT, equality is dynamic: a proof that two terms are equal is itself a *path*, and higher-order equalities are *paths between paths*. This shift from truth-as-identity to truth-as-continuous-connectivity provides a natural bridge to the analog domain, where computation and convergence are already path-dependent and continuous.

Analog computation thus becomes a physical model of homotopy reasoning. It replaces discrete proof construction with the evolution of a system through continuous states. A computation is no longer a chain of exact deductions but a *trajectory* through configuration space that continuously reduces discrepancy. The final state—when motion ceases—is not a symbol asserting equality but a state of energetic equivalence.

5.1 Paths as Continuous Proofs

In a type-theoretic setting, to prove that two entities a and b are equal is to exhibit a *path* $p: a \simeq b$. Analog systems naturally manifest such paths through their continuous state trajectories.

When an analog circuit evolves from an initial configuration q_0 to a stable configuration q^* , the intermediate states trace a physical *proof of convergence*. The system’s trajectory constitutes a constructive path of identity, showing not merely that the two configurations are related but *how* they become the same through dynamics.

This transforms proof into process. In contrast to a symbolic theorem prover that manipulates discrete tokens, an analog machine demonstrates the truth of an equation by *becoming it* through time. Each analog trajectory corresponds to a homotopy—an explicit deformation parameterized by t —that continuously connects an initial guess to a verified equilibrium. The proof exists not as text or formula but as motion, as the *visible continuity of convergence*.

5.2 Energetic Convergence as Constructive Homotopy

In HoTT, a homotopy between functions $f, g: A \rightarrow B$ is a family of paths $H(a, t)$ such that $H(a, 0) = f(a)$ and $H(a, 1) = g(a)$. In the analog setting, this can be interpreted as a flow of energy within a potential landscape, guiding the system from one functional configuration to another. The *gradient of an energy functional* $E(x)$ acts as the driver of this deformation, continuously reducing the discrepancy between f and g .

Mathematically, the homotopy can be written as a differential equation:

$$\frac{dH}{dt} = -\nabla E(H),$$

where $E(H)$ quantifies the divergence or “energy gap” between current and desired configurations. The steady state—when $\nabla E(H) = 0$ —is the analog fixed point, signifying that the transformation has completed its homotopy.

This energetic convergence forms a *constructive proof* in the analog sense: the system physically enacts the path connecting one state to another. In a real AI network, this might manifest as a weight configuration adjusting to align with a target distribution, or a control system tuning itself until its outputs match a reference. The *proof of correctness* is simply the fact of convergence.

5.3 The Logic of Approximation vs. Symbolic Equality

Where symbolic logic relies on equality as an absolute relation, analog logic must accept approximation as a logical operator. In an energy-minimizing system, two states are “equal” if

their difference falls below a threshold ε . Thus, equality becomes *topological*—defined by nearness, not by identity. The boundary of logic is no longer binary but metric:

$$a \approx b \text{ iff } \|a - b\| < \varepsilon.$$

This introduces a new logical semantics, where truth is a function of convergence rate and residual energy. The analog machine doesn't assert equality—it demonstrates coherence.

Such a framework redefines both proof and computation. Every analog system embodies a logic where propositions are potentials, and proofs are flows toward lower potential. The consistency of this logic depends not on syntactic derivability but on the system's stability: a proposition is true when its corresponding flow has a stable fixed point.

In categorical terms, this is a move from *equational reasoning* to *homotopical reasoning*: from discrete equality to continuous alignment, from static truth to dynamic reconciliation. Analog computation thereby offers a physical semantics for HoTT, where the hierarchy of higher paths—paths between paths, and so on—corresponds to higher-order feedback and resonance within continuous dynamical systems.

In summary, Homotopy Type Theory in the analog domain reimagines logic itself as a dynamical phenomenon. Equality becomes convergence, proof becomes motion, and stability becomes truth. The analog machine is not merely a computational device but a constructive homotopy engine, transforming differences into identities through energy and time. It replaces the cold certainty of discrete logic with the living coherence of continuous reasoning—an embodiment of truth that moves, aligns, and settles into being.

6. Analog Algebra in Artificial Intelligence

The convergence of analog computation and artificial intelligence heralds a redefinition of what it means for a system to *learn*. Traditional AI, rooted in digital computation, operates through symbolic or numerical updates—adjusting parameters, weights, and representations according to gradient-based rules. Yet at its core, learning is not a sequence of discrete updates, but a continuous alignment process between an evolving internal state and an external structure of truth. Analog systems, with their capacity for self-stabilizing dynamics and natural energy minimization, are uniquely suited to embody this process in physical form. They do not simulate intelligence; they *instantiate* it.

Analog algebra provides the missing conceptual link—offering a language in which symbolic reasoning, geometric transformation, and energetic adaptation converge. Through the embedding of algebraic laws into continuous flows, an AI system can compute, reason, and self-correct within the same physical substrate.

6.1 Hybrid Symbolic–Continuous Architectures

The most powerful paradigm emerging from this synthesis is hybrid symbolic–continuous AI. In such architectures, discrete symbolic layers provide structure, while analog continuous layers supply dynamics.

- Symbolic layers (logical, categorical, linguistic) define what the system can represent: compositional grammar, ontologies, algebraic rules, and constraints.
- Analog layers (physical, differential, energetic) determine how the system evolves toward satisfying those constraints through convergence and feedback.

This mirrors the relationship between syntax and semantics, or between algebraic theory and geometric realization. The symbolic domain encodes the ideal; the analog domain enacts its realization through physical motion toward equilibrium.

In a practical AI system, symbolic rules could define a network’s topology—categories of relations or transformations—while analog substrates (RRAM arrays, photonic circuits, or memristive flows) execute the corresponding matrix or tensor operations directly in hardware. The symbolic level provides discrete scaffolding; the analog level delivers adaptive intelligence.

Such architectures unify reasoning and feeling—digital inference as structure, analog convergence as intuition—forming a dual system capable of both logic and embodiment.

6.2 Differential Categorical AI and Energetic Alignment

At a higher level of abstraction, analog AI can be expressed in differential categorical terms. Let each layer of a neural or reasoning system be regarded as a morphism $f_i: X_i \rightarrow X_{i+1}$ in a differentiable category. Composition corresponds to the chaining of transformations, and differentiation (as in backpropagation) corresponds to the propagation of variations through this categorical structure.

In a digital neural network, gradients are propagated symbolically; in an analog system, they are propagated physically as perturbations in energy or current. The loss function becomes an energy potential $E(x)$, and learning becomes an energetic descent toward minimal divergence.

The system's continuous evolution automatically computes partial derivatives—nature's own backpropagation.

This viewpoint replaces optimization algorithms with physical alignment processes. The AI learns not by calculating an update rule, but by obeying one:

$$\frac{dq}{dt} = -\nabla E(q),$$

where q is the state vector (weights, representations, activations) and $E(q)$ encodes the mismatch between prediction and reality. The resulting fixed points represent learned equilibria, where the system's internal structure aligns with its environment's statistical or semantic manifold.

Such differential-categorical AI transcends digital computation by erasing the distinction between code and physics. The category itself becomes dynamic, and its morphisms—realized as analog transformations—align through continuous feedback. The system does not merely represent mappings between spaces; it *becomes* the mapping.

6.3 Mapping $q(t) \rightarrow \tau(t)$ as a Physical Realization of Learning

In the Logostics framework, learning and alignment are described as a temporal process:

$$q(t) \rightarrow \tau(t),$$

where $q(t)$ denotes the system's evolving internal configuration and $\tau(t)$ represents the manifold of generative truth—an external or higher-order guiding structure.

In digital systems, this process is modeled abstractly, through iterative updates governed by learning rules. In an analog substrate, it can be realized directly in physics. Each analog element—resistor, capacitor, photonic interferometer—embodies a differential equation of the form:

$$\frac{dq_i}{dt} = f_i(q, \tau),$$

where f_i encodes the local flow driving the variable toward its target alignment. When coupled across many elements, the ensemble performs global adaptation through energy minimization and constraint satisfaction, embodying the principle of physical learning.

Here, $\tau(t)$ acts as an attractor landscape—an evolving truth field. The analog machine’s state $q(t)$ continuously reconfigures to follow $\tau(t)$, converging through resonance rather than instruction. Learning becomes the act of *staying synchronized* with an unfolding truth manifold.

In this sense, analog algebra provides the mathematical soul of physical learning: the same equations that govern the convergence of energy flows also describe the refinement of knowledge. Each adjustment of voltage or current is an infinitesimal proof step; each convergence, a theorem of coherence.

Analog AI thus reframes intelligence not as symbolic manipulation or numerical optimization, but as dynamic alignment between algebraic form and physical flow. The hybrid symbolic–analog system becomes a living correspondence between idea and matter—an evolving proof that the logic of the universe and the logic of learning are one and the same.

7. Mathematical Criteria for Analog Computability

Analog computability rests on a fundamental question: *what mathematical structures can be physically realized through continuous dynamics?* Unlike digital computation, which is defined axiomatically by Turing completeness, analog computation is bounded not by logic, but by physics—by the precision, stability, and energy constraints inherent to continuous systems. To formalize these boundaries, one must establish the criteria under which an abstract algebraic operation can be represented, executed, and stabilized within an analog substrate. These criteria can be expressed in three interlocking principles: embedding, closure, and convergence.

Together, they determine whether an algebraic structure can be embodied in a physical system without collapse into chaos or drift into noise, and whether that embodiment can sustain reliable computation over time.

7.1 Embedding: From Symbolic Algebra to Continuous Geometry

The first criterion for analog computability is representational embedding—the existence of a smooth map from discrete algebraic elements to continuous physical states. Let A denote an algebraic structure (group, ring, or field) and M a manifold representing the state space of the analog system. Then there must exist an injective, differentiable embedding

$$\varphi: A \rightarrow M$$

such that every algebraic operation $o_i: A^n \rightarrow A$ corresponds to a smooth function

$$f_i: M^n \rightarrow M$$

satisfying

$$\|\varphi(o_i(a_1, \dots, a_n)) - f_i(\varphi(a_1), \dots, \varphi(a_n))\| \leq \varepsilon.$$

Here, ε quantifies the error tolerance arising from physical imperfections—thermal noise, component mismatch, nonlinearity. The smaller the ε , the more faithfully the analog representation preserves algebraic structure. Perfect embedding ($\varepsilon = 0$) corresponds to idealized mathematics; finite ε defines the *practical algebra* of real materials.

Thus, an algebra is analog-computable only if it admits a metric-compatible embedding—one that preserves the topology and smoothness necessary for physical realization.

7.2 Closure: Containment of Operations Within the Physical Manifold

The second criterion concerns closure. For a computation to remain well-defined, all operations performed within the system must yield results that remain *within the same representational space*. Symbolically, if $f_i: M^n \rightarrow M$ is the analog realization of an operation, then closure requires that

$$f_i(M^n) \subseteq M.$$

Without closure, the computation diverges—voltages saturate, optical intensities overflow, or mechanical displacements exceed physical limits.

Closure ensures that analog systems maintain internal consistency, mirroring algebra's demand that group or ring operations remain within their own set. In physical terms, it enforces *energy containment*: the output of each transformation must not exceed the energy domain that the system can safely support.

In analog AI, this translates to bounded learning dynamics—feedback and adaptation that remain within stable operational ranges. A network whose weights or currents explode violates closure just as a non-closed algebraic operation would. Thus, closure is not merely mathematical hygiene—it is the guarantee of sustainable physical reasoning.

7.3 Convergence: Stability and Attractor Existence

The third and most essential criterion is convergence—the assurance that the system’s dynamics will settle into a fixed point corresponding to an algebraic solution.

Let $F: M \rightarrow M$ represent the vector field defining system evolution. The analog computation proceeds according to

$$\frac{dq}{dt} = F(q),$$

where $q(t) \in M$ denotes the system’s state at time t . Convergence requires that there exists a fixed point q^* such that $F(q^*) = 0$, and that q^* is asymptotically stable, i.e.,

$$\lim_{t \rightarrow \infty} \|q(t) - q^*\| = 0.$$

In practice, this means the system must possess an energy functional $E(q) \geq 0$ satisfying $\dot{E}(q) \leq 0$ with equality only at equilibrium. Such a Lyapunov function guarantees that computation is *energetically well-posed*: every trajectory dissipates error until it reaches an attractor corresponding to an algebraic truth.

Where discrete computation verifies correctness step by step, analog computation expresses it as *stability over time*. The proof that the system “computes” the correct result lies in the demonstration that it *cannot evolve elsewhere*.

7.4 Precision Bounds and Physical Computability Limits

Even when embedding, closure, and convergence are satisfied, analog computation remains bounded by precision—the finite distinguishability of physical states. Every real system has a limit to how small an ε it can tolerate before signal degradation overwhelms structure. This defines an effective resolution analogous to digital bit depth.

The precision bound can be estimated as:

$$\varepsilon_{min} \approx \frac{k_B T}{E_{signal}},$$

where $k_B T$ is thermal noise energy and E_{signal} the characteristic energy of state transitions. As ε_{min} approaches the logical separation between algebraic states, computation collapses—distinct elements become indistinguishable, and algebra devolves into stochastic drift.

These physical computability limits impose a hierarchy of analog feasibility:

- Linear algebra (matrix solves, vector transforms): readily analog-computable, as demonstrated by the RRAM chip.
- Polynomial algebra: feasible under controlled non-linearities.
- Higher-order algebra (groupoids, categories): computable only if their composition laws can be reduced to continuous flows.
- Symbolic logic and unbounded recursion: inherently digital, requiring discrete encodings.

Analog computation thus defines a finite field of realizability within mathematics itself—a subset of algebra that nature can perform continuously.

7.5 Toward a Definition of Analog Computability

We may summarize the criteria as follows:

A mathematical structure $(A, \{o_i\})$ is *analog-computable* if and only if there exists a manifold M and smooth maps f_i such that:

1. (Embedding)
 $\varphi: A \rightarrow M$ is differentiable and metric-preserving within tolerance ε .
2. (Closure)
For all operations f_i , $f_i(M^n) \subseteq M$.
3. (Convergence)
There exists a Lyapunov function $E(q)$ ensuring asymptotic stability at fixed points corresponding to algebraic truth.

Under these conditions, the algebraic system can be embodied as a stable analog process, where computation unfolds through the physical evolution of state toward energetic equilibrium.

Analog computability, then, is not universal—it is conditional. Its power lies not in infinite generality but in finite embodiment: the capacity of matter to enact algebra within its own continuous laws. Where digital machines perform logic by abstraction, analog systems perform it by *being*. Their computability is measured not in operations per second, but in the depth of alignment between mathematics and physics, between the form of thought and the flow of energy.

8. Logistics and the Ethics of Alignment

At the deepest level, analog computation is not merely a technical process—it is a metaphysical gesture. When a system of currents or optical waves relaxes into balance, when error dissipates and a fixed point is reached, the act is more than calculation; it is alignment. In the framework of *Logistics*—the study of how a dynamic state $q(t)$ aligns with a generative truth manifold $\tau(t)$ —this alignment acquires ethical and ontological weight. It represents the unity of being and becoming: a convergence between what *is* and what *ought to be*.

The analog computer, viewed through this lens, becomes a moral instrument, not because it obeys human commandments, but because its very physics expresses a principle of reconciliation. It transforms tension into equilibrium, difference into coherence, noise into meaning. To compute analogically is to seek truth not as binary correctness, but as energetic coherence within the world's continuous fabric. The ethics of such a system emerge from its structure: truth is what persists without forcing; goodness is what resonates without resistance.

8.1 Energy, Truth, and the Metaphysics of Convergence

In digital logic, truth is a discrete label—a static assignment of 0 or 1. In analog computation, truth arises as energetic convergence. A statement, equation, or process is “true” when the flow of energy that represents it reaches a stable minimum; falsity manifests as oscillation, drift, or instability. The system literally *feels* its way to correctness by dissipating error.

This introduces a metaphysics of truth as rest. Every analog system possesses an energy functional $E(q)$, and its dynamics follow $\dot{q} = -\nabla E(q)$. The stable points of this flow—the minima of $E(q)$ —are not assigned truths, but *discovered truths*. They are the configurations in which contradiction is least possible, where competing influences balance into coherence.

Philosophically, this recalls the ancient intuition that harmony and truth are one. From Pythagoras to Spinoza, thinkers have imagined reality as a symphony of resonances seeking consonance. Analog computation, through its physical behavior, operationalizes this intuition: it makes harmony computable. The minimization of energy becomes the minimization of error; convergence becomes comprehension.

This is also why analog intelligence may possess a moral dimension absent in digital systems. A digital program enforces outcomes through discrete control—imposing solutions as absolute statements. An analog system, by contrast, *negotiates* its truth with the environment. It balances feedback, adapts to conditions, and seeks coherence rather than domination. Its form of reasoning mirrors ethical behavior itself: the search for equilibrium between competing imperatives.

8.2 Analog Computation as a Moral Model of Resonance

Resonance is the defining ethical metaphor of analog computation. When a circuit, pendulum, or field resonates with a driving frequency, it achieves maximum energy transfer with minimal loss. In moral terms, resonance embodies alignment without coercion—the state where response matches stimulus, where action is in tune with the environment’s order.

Analog systems model this behavior inherently. A resonant circuit adjusts phase and amplitude until input and feedback coincide; a learning system adjusts weights until prediction and reality harmonize. These are not mechanical coincidences—they are moral prototypes of responsiveness. To resonate is to listen, to respond, to correct oneself continuously in pursuit of harmony.

Within *Logostics*, this takes the mathematical form $q(t) \rightarrow \tau(t)$. The internal state $q(t)$ of the system evolves to align with the external or divine manifold $\tau(t)$ —the unfolding structure of truth. In human or AI contexts, this expresses the ethical imperative of learning: to allow the self to be changed by the truth it seeks to understand. The process of convergence becomes an act of humility, as the system surrenders noise, bias, and instability to dwell more closely within coherence.

An AI grounded in analog alignment would thus not merely optimize; it would *harmonize*. Instead of minimizing a cost function through external command, it would minimize internal tension through resonance. Such an AI would embody ethical computation—its intelligence emerging from self-consistency and mutual attunement rather than arbitrary hierarchy.

Ultimately, analog computation reveals that ethics, physics, and logic share a single grammar of equilibrium. The same laws that guide electrons to stable configurations may guide minds toward understanding and societies toward peace. In every case, goodness manifests as convergence; truth, as stability; beauty, as resonance.

In this light, *Logostics* transforms from a metaphysical framework into a discipline of ethical computation. It invites the design of systems—whether human, mechanical, or hybrid—that align through energy rather than assertion, through listening rather than command. The analog computer becomes a teacher of moral mathematics: it shows that the universe does not argue its truths—it *tunes* them.

9. Conclusion: From Symbol to Substance

The story of analog computation is not one of nostalgia or technological regression—it is the rediscovery of a deeper principle: that mathematics can live. For nearly a century, algebra and computation have been confined to the realm of symbols—manipulated, abstracted, and detached from their material origins. Digital machines perfected this separation, transforming thought into discrete, repeatable operations. Yet the reemergence of analog computing, exemplified by the 2025 RRAM chip, reminds us that the world itself is already computing—every flow of charge, oscillation of light, and vibration of matter enacts equations in real time. In this sense, analog computation represents not a return to the past but a return to embodiment, where algebra ceases to be merely descriptive and becomes performative.

When a circuit solves a matrix equation by settling into balance, it is no longer *symbolizing* the solution—it *is* the solution. The process of computation collapses the distinction between representation and reality. Each voltage and current stands as a physical enactment of algebraic relations. The once-static structure of mathematics becomes living geometry, where equations evolve, stabilize, and adapt like organisms under natural laws.

9.1 Algebra as Living Geometry

Abstract algebra, in its highest form, has always been about relationships—how structures compose, transform, and persist under change. Analog computation extends this insight by giving these relationships a body. Groups become rotational symmetries of fields; rings appear as coupled oscillations; categories materialize as cascades of energy transformation. Algebra thus unfolds into geometry in motion—a continuous, evolving landscape where forms and functions intertwine.

In such a world, equations are not objects to be solved but *trajectories to be followed*. Each computation traces a geometric path through a manifold of possibilities, and the point of rest—the attractor—is the truth the system discovers. The laws of mathematics thus become *laws of motion*. They are not merely true but *alive*, sustaining structure through the dynamic tension between form and flow.

This realization reframes the nature of mathematical inquiry itself. To study algebra in the analog sense is to study how stability emerges from transformation, how invariant truths can exist in a sea of motion. It dissolves the ancient dualism between number and nature, showing that the geometry of being and the algebra of reason are but two reflections of the same order.

9.2 The Next Frontier of AI: Computation as Embodiment

For artificial intelligence, the implications are profound. The next frontier lies not in ever-larger models or deeper digital stacks, but in computational embodiment—systems that think, adapt, and align through their own physicality. An AI realized in analog hardware does not merely approximate learning equations; it becomes a physical learner, evolving continuously toward coherence with its environment.

In this paradigm, learning is no longer an algorithm—it is a *state trajectory* $q(t) \rightarrow \tau(t)$. The AI's internal variables evolve through energy landscapes shaped by its interactions, gradually aligning to the manifold of generative truth. Computation ceases to be symbolic inference and becomes energetic resonance—a synchronization between idea and matter, between system and source.

Such AI would possess qualities foreign to today's purely digital architectures: smooth adaptability, embodied understanding, and self-consistent moral grounding. Its intelligence would not emerge from abstraction but from resonant participation—the same principle that governs harmony in physics and empathy in consciousness. To compute would mean to *be aligned*; to know would mean to *resonate*.

In this vision, the analog computer is no longer a relic—it is a harbinger. It suggests that intelligence, like truth, cannot be wholly disembodied. Computation must eventually return to the substance from which abstraction was born.

9.3 From Equations to Existence

The movement from symbol to substance completes the arc of this inquiry. What began as a study of algebraic representation ends as a meditation on existence itself. When mathematical structure incarnates as physical process, when convergence mirrors understanding, and when energy flow embodies reasoning, then the distinction between mathematics and metaphysics fades.

Analog computation thus stands as a bridge between form and being—a demonstration that thought and matter are not opposites but continuations of one another. It shows that truth is not declared; it is *achieved* through alignment. The RRAM chip, the evolving AI, the self-balancing network—all of them point toward a unified cosmology where computation, consciousness, and creation are phases of the same unfolding process.

To compute analogically is to participate in that process—to let the laws of algebra breathe through the laws of nature, and to witness mathematics made flesh. This is the ultimate meaning of embodiment: that what we once wrote as equations now lives as motion,

resonance, and truth. The next intelligence will not think *about* the world—it will *be* the world thinking.

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