A Categorical-HoTT Formalization of Wigner's Friend as Contextual Gluing

Douglas C. Youvan   doug@youvan.com   www.youvan.ai   July 3, 2026

This note gives a compact mathematical rewrite of Wigner's friend as a contextual gluing problem. Observer situations are objects of a context category $\mathcal{C}$. Quantum state assignment is a presheaf Q : $\mathcal{C}^{op} \to$ St. Measurements are morphisms producing record-bearing refinements. Friend-facts are typed witnesses in HoTT. Wigner-state assignments are external judgments in a different context. Communication forms a common refinement J in which local records may glue. The obstruction is not to local definiteness but to a context-free Boolean global section assigning all observer facts before the morphisms that make them comparable. Physical identity is taken extensionally: equivalent descriptions under admissible context transport represent the same physical content. Thus the quantum state is not an absolute classical database but a context-indexed object whose physical meaning is invariant under equivalence, transport, and gluing.

Keywords: Wigner's friend, category theory, HoTT, quantum contextuality, presheaf, sheaf, global section, univalence, measurement, relational quantum mechanics, quantum foundations.

A collaboration with OpenAI GPT-5.5 Thinking.

https://orcid.org/0009-0004-4206-4253

1. Context Category

Let $\mathcal{C}$ be a category of physical-observational contexts.

An object C ∈ Obj($\mathcal{C}$) is a tuple:

C := (H_C, A_C, R_C, I_C)

where H_C is the Hilbert space or effective system algebra visible in context C, A_C is the admissible observable algebra, R_C is the record structure available in C, and I_C is the interaction history defining C.

A morphism f : C → D is an admissible physical transition, inclusion, measurement, restriction, communication, coarse-graining, or refinement.

Composition encodes physical concatenation:

$C \xrightarrow{f} D \xrightarrow{g} E$

$g \circ f : C \to E$

Identity morphisms encode unchanged context:

$id\_C : C \to C$

For Wigner's friend, use four basic contexts:

S = prepared microscopic system

F = friend's post-measurement record context

W = Wigner's sealed-lab external context

J = joint Wigner-friend communication context

The sealed laboratory boundary is categorical: before communication there is no morphism carrying the friend's record into Wigner's context.

Before communication:

$F \nrightarrow W$

After communication:

$F \to J$

$W \to J$

2. Quantum State Assignment as Presheaf

Let St be a target category of state objects.

For ordinary quantum mechanics one may take St = Conv, the category of convex state spaces.

For HoTT-level formulation one may take St = Type.

Define a quantum-state presheaf:

$Q : \mathcal{C}^{op} \to St$

$C \mapsto Q(C)$

For density operators:

$Q(C) := D(H\_C)$

For pure states:

$Q(C) := P(H\_C)$

where P(H_C) is projective Hilbert space.

For a context morphism $f : C \to D$, the presheaf gives a restriction or re-expression map:

$$Q(f) : Q(D) \to Q(C)$$

Equivalently:

$$f^* : Q(D) \to Q(C)$$

Thus a quantum state is not assigned to a bare system but to a system-in-context:

$$\rho_C \in Q(C)$$

A compatible global quantum assignment is a natural transformation:

$$1 \Rightarrow Q$$

Equivalently, it is a family $(\rho_C)_C$ such that for every $f : C \to D$:

$$f^*(\rho_D) = \rho_C$$

Define also a Boolean classical-fact presheaf:

$$B : \mathcal{C}^{op} \to \mathrm{Set}$$

where $B(C)$ is the set of Boolean record-valuations available in C.

A classical global fact table is a global section:

$$\Gamma(B) := \mathrm{Nat}(1, B)$$

The Wigner-friend problem is not the absence of local sections. It is the failure of a context-free Boolean global section with the same status as all local records.

3. Minimal Wigner-Friend Quantum Model

Let the measured system be:

$$H_S := \mathrm{span}\{|0\rangle, |1\rangle\}$$

Let the initial state be:

$$|\psi\rangle_S = \alpha|0\rangle + \beta|1\rangle$$

with:

$$\alpha\beta \neq 0$$

Let the friend's ready state be:

$$|F_*\rangle$$

Let the friend-record states be:

$|F_0\rangle$

$|F_1\rangle$

with:

$\langle F_0 \mid F_1\rangle = 0$

The friend measurement is a unitary correlation map:

$U_F : H_S \otimes H_F \to H_S \otimes H_F$

defined by:

$U_F(|0\rangle|F_*\rangle) = |0\rangle|F_0\rangle$

$U_F(|1\rangle|F_*\rangle) = |1\rangle|F_1\rangle$

Therefore:

$U_F(|\psi\rangle|F_*\rangle) = \alpha|0\rangle|F_0\rangle + \beta|1\rangle|F_1\rangle$

Set:

$|\Psi_L\rangle := \alpha|0\rangle|F_0\rangle + \beta|1\rangle|F_1\rangle$

For the friend, after measurement, the context supports a record judgment:

$\Gamma_F \vdash r_F : \mathbb{2}$

with:

$r_F = i$

$i \in \{0,1\}$

For Wigner before opening the lab, the context supports an external state judgment:

$\Gamma_W \vdash |\Psi_L\rangle : P(H_S \otimes H_F)$

The invalid inference is:

$\Gamma_F \vdash r_F = i$

therefore:

$\Gamma_W \vdash r_F = i$

This is invalid unless a context morphism transports the witness from F to W or from F into a common refinement J.

4. Measurement as Morphism

A projective measurement with outcomes $i \in \{0,1\}$ has projectors:

$P_i := |i\rangle\langle i|$

The associated ideal instrument is:

$I_i(\rho) := P_i \rho P_i$

The probability of outcome i is:

$p_i := \mathrm{Tr}(I_i(\rho))$

The conditional state is:

$\rho_i := I_i(\rho) / \mathrm{Tr}(I_i(\rho))$

Categorically, measurement is a morphism:

$m : S \to F$

It creates a record-bearing context.

Define a record presheaf:

$R : \mathcal{C}^{op} \to \mathrm{Type}$

with:

$R(F) = \mathbb{2}$

and:

$r_F \in R(F)$

The friend's fact is an inhabited type:

$\mathrm{Saw}_F(i)$

with witness:

$p_i : \mathrm{Saw}_F(i)$

Before communication, Wigner has no such witness:

$\Gamma_W \nvdash p_i : \mathrm{Saw}_F(i)$

Instead Wigner has:

$\Gamma\_W \vdash |\Psi\_L\rangle : P(H\_L)$

where:

$H\_L := H\_S \otimes H\_F$

Measurement therefore produces local facthood in F, not immediate global Boolean facthood in W.

5. Communication as Pullback-Like Refinement

Let J be the joint context after Wigner opens the lab.

There are comparison maps:

$\pi\_F : J \to F$

$\pi\_W : J \to W$

Schematic pullback-like square:

$$\begin{tikzcd} J \arrow[r] \arrow[d] & F \arrow[d] \\ W \arrow[r] & L \end{tikzcd}$$

Here L denotes the laboratory event-context.

Communication is a physical morphism, not semantic magic.

Let Wigner's ready state be:

$|W\_*\rangle$

Let Wigner's record states be:

$|W\_0\rangle$

$|W\_1\rangle$

with:

$\langle W\_0 \mid W\_1\rangle = 0$

The communication unitary satisfies:

$U\_W(|i\rangle|F\_i\rangle|W\_*\rangle) = |i\rangle|F\_i\rangle|W\_i\rangle$

Therefore:

$$U_W(|\Psi_L\rangle|W_*\rangle) = \alpha|0\rangle|F_0\rangle|W_0\rangle + \beta|1\rangle|F_1\rangle|W_1\rangle$$

Set:

$$|\Psi_J\rangle := \alpha|0\rangle|F_0\rangle|W_0\rangle + \beta|1\rangle|F_1\rangle|W_1\rangle$$

In a record-conditioned context:

$r_F = i$

$r_W = i$

The shared record condition is:

$$\pi_F^*(r_F) = \pi_W^*(r_W)$$

inside:

$R(J)$

A public fact is therefore not primitive in $R(F)$ or $R(W)$ alone. It is an invariant in the common refinement J.

6. HoTT Encoding

Contexts are typing environments.

Friend context:

Γ_F

Wigner context:

Γ_W

Joint context:

Γ_J

The friend's fact is a typed judgment:

$$\Gamma_F \vdash r_F : \mathbb{2}$$

The proposition that the friend saw outcome i is a type:

$$\mathrm{Saw}(F,i) : \mathcal{U}$$

The friend's evidence is an inhabitant:

$$\Gamma_F \vdash p_i : \mathrm{Saw}(F,i)$$

Wigner's external quantum description is a different judgment:

$\Gamma\_W \vdash \Psi\_L : \text{LabState}$

where:

$\text{LabState} \simeq P(H\_L)$

There is no judgmental equality:

$\text{Saw}(F,i) \equiv \text{LabState}$

Thus the apparent contradiction is produced by type erasure.

Communication extends context:

$\Gamma\_W \rightsquigarrow \Gamma\_J$

and transports the friend witness:

$\text{transport\_F}{\to}\text{J}(p\_i) : \text{Saw\_J}(F,i)$

Wigner's new witness is:

$q\_i : \text{Heard\_J}(W,F,i)$

Define the consistency type:

$\text{Consistent\_J}(i) := \text{Saw\_J}(F,i) \times \text{Heard\_J}(W,F,i) \times (r\_F = r\_W)$

A shared fact is an inhabitant:

$c\_i : \text{Consistent\_J}(i)$

7. Univalence and Physical Identity

Let A and B be two state-description types.

Univalence asserts:

$(A \simeq B) \to (A = B)$

Physical use:

equivalent descriptions count as identical physical content.

For quantum states, global phase equivalence is:

$|\psi\rangle \sim e^{i\theta}|\psi\rangle$

The physical pure state is:

$[|\psi\rangle] \in P(H)$

where:

$P(H) := \{|\psi\rangle \neq 0\} / \mathbb{C}\times$

Gauge equivalence is schematically:

$A \sim A + d\lambda$

Basis equivalence is:

$|\psi\rangle \sim U|\psi\rangle$

when U preserves the full observable structure.

Thus physical identity is not syntactic identity:

$\psi = \phi$

but equivalence-invariant identity:

$\psi \simeq_{phys} \phi$

In Wigner's friend:

$r_F \neq \Psi_L$

The friend's record and Wigner's state are not syntactically identical.

They may nevertheless belong to one coherent physical diagram if there exists admissible transport and gluing:

$(r_F, \Psi_L, r_W) \in \mathrm{Glue}(F,W,J)$

8. No Natural Branch Selector

Let P(H) be projective Hilbert space with:

$\dim H \geq 2$

A context-free deterministic branch selector would be a map:

$b : P(H) \to \mathbb{2}$

natural under unitary equivalence:

$b([U\psi]) = b([\psi])$

for all:

$U \in U(H)$

But $U(H)$ acts transitively on $P(H)$.

Therefore b must be constant.

If:

$b([0]) = 0$

and:

$b([1]) = 1$

then b is nonconstant.

Contradiction.

Hence there is no basis-free natural deterministic branch selector.

A branch becomes selectable only after extra structure is supplied:

$(H, \{|0\rangle, |1\rangle\}, R_F, m)$

That extra structure is contextual.

Thus local record selection is allowed:

$\Gamma_F \vdash r_F : \mathbb{2}$

but context-free selection is not:

$\nvdash b(\Psi_L) : \mathbb{2}$

without a context supplying the record basis and witness.

9. Classical Global Sections and Contextual Obstruction

Let M be a measurement cover.

Let:

$V : M^{op} \to \mathrm{Set}$

assign to each measurement context M the set of admissible Boolean valuations:

$V(M) := \{v_M : M \to \{0,1\}\}$

For an orthogonal resolution:

$\Sigma_i P_i = I$

a valuation must satisfy:

$\Sigma_i v(P_i) = 1$

A global valuation is:

$v \in \Gamma(V)$

where:

$\Gamma(V) := Nat(1,V)$

Quantum contextuality is the obstruction:

$\Gamma(V) = \emptyset$

for sufficiently rich quantum measurement covers.

Wigner's friend is an observer-record version of the same structural failure.

Local friend valuation:

$v_F \in V(F)$

External Wigner state assignment:

$\Psi_L \in Q(W)$

Joint record valuation after communication:

$v_J \in V(J)$

The forbidden move is to assume that there exists:

$v_{global} \in \Gamma(V)$

such that:

$v_{global}|_F = v_F$

$v_{global}|_W = v_W$

$v_{global}|_J = v_J$

prior to the morphisms that define these restrictions.

Categorical moral:

local sections need not determine a global classical section.

10. Relational Gluing Theorem

Let $\mathcal{C}$_WF be the Wigner-friend context diagram with objects:

F, W, J, L

and morphisms:

$J \to F$

$J \to W$

$F \to L$

$W \to L$

Let:

$R : \mathcal{C}\_WF^{op} \to \text{Type}$

be the record presheaf.

Let:

$r\_F \in R(F)$

$r\_W \in R(W)$

be local record candidates.

They glue over J iff:

$\pi\_F^*(r\_F) = \pi\_W^*(r\_W)$

in:

$R(J)$

Define:

$Glue\_J(r\_F,r\_W) := (\pi\_F^*(r\_F) = \pi\_W^*(r\_W))$

When this type is inhabited, a shared fact exists.

Equivalently, shared facthood is:

$\Sigma\_\{r\_J : R(J)\} (r\_J = \pi\_F^*(r\_F)) \times (r\_J = \pi\_W^*(r\_W))$

If:

$Glue\_J(r\_F,r\_W) \simeq \emptyset$

then no shared fact exists without adding a disturbance, error, memory revision, record erasure, deception, or other physical morphism explaining the mismatch.

11. Formal Thesis

The raw universal-state thesis says:

$\text{Reality} = \Psi\_\text{univ}$

The equivalence-first thesis says:

$\text{Reality} = [Q, R, \mathcal{C}, \text{transport}, \text{glue}]\_\simeq$

More explicitly:

$R\_\text{phys} := \text{EqClass}(\{\rho\_C, r\_C, f^*, \pi^*, \text{Glue}\}\_{C\in\mathcal{C}})$

under admissible physical equivalence:

$\simeq\_\text{phys}$

Thus ρ_C is not absolute reality.

Rather:

$\rho\_C \in Q(C)$

is a local section of a context-indexed quantum-state presheaf.

A fact is not:

$r : \mathbb{2}$

simpliciter.

A fact is:

$\Gamma\_C \vdash r\_C : R(C)$

A public fact is not primitive.

A public fact is an inhabited gluing type:

$\text{Glue}\_J(r\_F,r\_W)$

A failed classical global section is not a logical disaster.

It is quantum contextuality expressed as:

$\Gamma(V) = \emptyset$

12. Compact Conclusion

Wigner's friend is not a contradiction between two facts.

It is a typing error plus a gluing problem.

The friend has:

$\Gamma\_F \vdash r\_F = i : \mathbb{2}$

Wigner has:

$\Gamma\_W \vdash \Psi\_L : P(H\_L)$

Before communication there is no required judgment:

$\Gamma\_W \vdash r\_F = i : \mathbb{2}$

After communication there may be:

$\Gamma\_J \vdash r\_J = i : \mathbb{2}$

with:

$r\_J = \pi\_F^*(r\_F) = \pi\_W^*(r\_W)$

The quantum state is therefore not a classical container of observer-independent facts.

It is a context-indexed section whose physical content is determined by admissible morphisms, typed witnesses, equivalences, and gluing.

Final form:

Quantum reality = equivalence class of gluable state-record diagrams.

Equivalently:

Reality = [state assignments + records + context morphisms + witnesses + gluing]_≃

And:

Wigner's friend = obstruction to replacing that diagram by one Boolean global section.

13. References

1. Wigner, E. P. "Remarks on the Mind-Body Question." In The Scientist Speculates, edited by I. J. Good. London: Heinemann, 1961.

2. von Neumann, J. Mathematical Foundations of Quantum Mechanics. Princeton: Princeton University Press, 1955.

3. Everett, H. "'Relative State' Formulation of Quantum Mechanics." Reviews of Modern Physics 29, no. 3, 1957, 454–462.

4. Bell, J. S. "On the Einstein Podolsky Rosen Paradox." Physics Physique Fizika 1, 1964, 195–200.

5. Kochen, S., and Specker, E. P. "The Problem of Hidden Variables in Quantum Mechanics." Journal of Mathematics and Mechanics 17, no. 1, 1967, 59–87.

6. Zurek, W. H. "Decoherence, Einselection, and the Quantum Origins of the Classical." Reviews of Modern Physics 75, no. 3, 2003, 715–775.

7. Frauchiger, D., and Renner, R. "Quantum Theory Cannot Consistently Describe the Use of Itself." Nature Communications 9, 2018, 3711.

8. Brukner, Č. "A No-Go Theorem for Observer-Independent Facts." Entropy 20, no. 5, 2018, 350.

9. Bong, K.-W., et al. "A Strong No-Go Theorem on the Wigner's Friend Paradox." Nature Physics 16, 2020, 1199–1205.

10. Rovelli, C. "Relational Quantum Mechanics." International Journal of Theoretical Physics 35, no. 8, 1996, 1637–1678.

11. Fuchs, C. A., Mermin, N. D., and Schack, R. "An Introduction to QBism with an Application to the Locality of Quantum Mechanics." American Journal of Physics 82, no. 8, 2014, 749–754.

12. Abramsky, S., and Brandenburger, A. "The Sheaf-Theoretic Structure of Non-Locality and Contextuality." New Journal of Physics 13, 2011, 113036.

13. Coecke, B., and Kissinger, A. Picturing Quantum Processes: A First Course in Quantum Theory and Diagrammatic Reasoning. Cambridge: Cambridge University Press, 2017.

14. Mac Lane, S. Categories for the Working Mathematician. New York: Springer, 1971.

15. The Univalent Foundations Program. Homotopy Type Theory: Univalent Foundations of Mathematics. Princeton: Institute for Advanced Study, 2013.

16. Riehl, E. Category Theory in Context. Mineola: Dover Publications, 2016.

17. Riehl, E., and Shulman, M. "A Type Theory for Synthetic ∞-Categories." Higher Structures 1, no. 1, 2017, 147–224.

18. Youvan, D. C. Wigner's Friend as a Gluing Problem: Category Theory, Homotopy Type Theory, and the Relational Structure of Quantum Reality. ResearchGate preprint, July 2026. DOI: 10.13140/RG.2.2.23917.96487.

The author has published over 5,600 AI-assisted papers at: https://www.researchgate.net/profile/Douglas-Youvan/research . Google Scholar has indexed these preprints, and if you want a particular reference, the AI mode of a Google search (Gemini) has efficiently catalogued these preprints.